Straight lines

P3.1 · Pure mathematics · revision notes

Equations of straight lines in the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$, gradients, and the conditions for two lines to be parallel or perpendicular. It includes straight-line models in context. Line questions often combine with circles, intersection points and areas of triangles.

Key facts

- $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- $y = mx + c$ and $ax + by + c = 0$
- Parallel: $m_1 = m_2$
- Midpoint $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
- $y - y_1 = m(x - x_1)$
- Gradient of $ax + by + c = 0$ is $-\dfrac{a}{b}$
- Perpendicular: $m_1 m_2 = -1$
- Distance $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

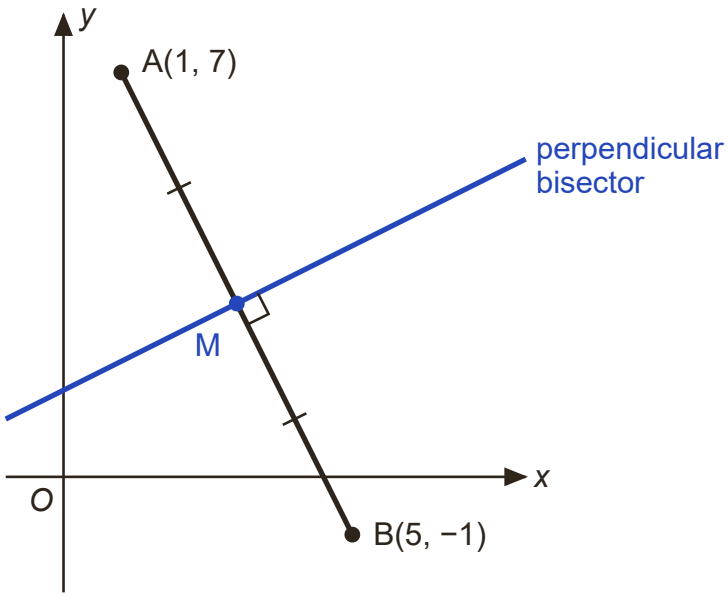
Notes

Gradient and equations

- Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- $y = mx + c$: gradient m, y-intercept $(0, c)$
- $y - y_1 = m(x - x_1)$: the line through (x_1, y_1) with gradient m. This is usually the quickest form to write down.
- $ax + by + c = 0$: exam questions often ask for integers a, b and c. Its gradient is $-\dfrac{a}{b}$
- Example: the line through $(2, -1)$ with gradient $\dfrac{3}{4}$ is $y + 1 = \dfrac{3}{4}(x - 2)$, so $4y + 4 = 3x - 6$, i.e. $3x - 4y - 10 = 0$

Parallel and perpendicular lines

- Parallel lines have equal gradients.
- Perpendicular lines: $m_1 \times m_2 = -1$, so the perpendicular gradient is $-\dfrac{1}{m}$ (flip and change the sign). E.g. $\dfrac{2}{3}$ becomes $-\dfrac{3}{2}$
- The perpendicular bisector of AB passes through the midpoint of AB and is perpendicular to AB



The perpendicular bisector goes through the midpoint M(3, 3) and has gradient −1/m: here ½, because AB has gradient −2.

Points, lengths and areas

- Midpoint: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$. Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Two lines meet where their equations are both true: solve them simultaneously.
- A line meets the x-axis where $y = 0$ and the y-axis where $x = 0$
- For the area of a triangle, sketch it and look for a side along an axis or a right angle, then use $\dfrac{1}{2} \times$ base $\times$ height.

Straight-line models

- In a linear model the gradient is the rate of change (e.g. cm per kg) and the intercept is the value when the variable is 0 (e.g. a fixed charge).
- A linear model is only valid over a sensible range of values; predictions far outside the data are unreliable.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find the equation of a line through two points

4 marks | Grade D

1. Find the gradient.
2. Use $y - y_1 = m(x - x_1)$ with one of the points.
3. Rearrange into the form asked for, with integer coefficients.

Example. The line l passes through the points $A(-3, 2)$ and $B(5, 8)$. Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

Model answer.

$$m = \frac{8 - 2}{5 - (-3)} \quad \text{(M1)}$$

$$m = \frac{3}{4} \quad \text{(A1)}$$

$$y - 2 = \frac{3}{4}(x + 3) \quad \text{(M1)}$$

$$3x - 4y + 17 = 0 \quad \text{(A1)}$$

Find a perpendicular bisector

5 marks | Grade B

1. Find the midpoint of the segment.
2. Find the gradient of the segment, then the perpendicular gradient.
3. Use $y - y_1 = m(x - x_1)$ through the midpoint.

Example. The points A and B have coordinates $(1, 7)$ and $(5, -1)$. Find an equation of the perpendicular bisector of AB, giving your answer in the form $ax + by + c = 0$

Model answer.

Midpoint $(3, 3)$ (B1)

Gradient of $AB = \frac{-1 - 7}{5 - 1} = -2$ (M1)

Perpendicular gradient $\frac{1}{2}$ (A1)

$$y - 3 = \frac{1}{2}(x - 3) \quad \text{(M1)}$$

$$x - 2y + 3 = 0 \quad \text{(A1)}$$

Intersections and the area of a triangle

7 marks **Grade A**

1. Find the equation of the missing line.
2. Solve simultaneously to find where the lines meet.
3. Find the axis intercepts.
4. Sketch the triangle and use $\frac{1}{2} \times$ base $\times$ height.

Example. The line l_1 has equation $y = 2x - 2$. The line l_2 is perpendicular to l_1 and passes through the point $B(11, 0)$.

(a) Find an equation for l_2.

(b) Find the coordinates of the point P where l_1 and l_2 meet.

(c) The line l_1 crosses the x-axis at A. Find the area of triangle APB

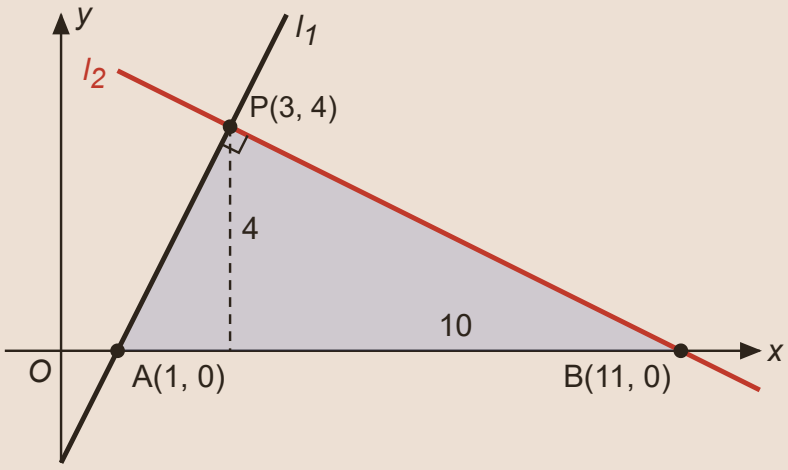
Model answer.

(a) Gradient of l_2 is $-\frac{1}{2}$ (M1)

$y = -\frac{1}{2}(x - 11)$, i.e. $x + 2y - 11 = 0$ (A1)

(b) $2x - 2 = -\frac{1}{2}x + \frac{11}{2}$ (M1)

$P(3, 4)$ (A1)



(c) $A(1, 0)$ (B1)

Area $= \frac{1}{2} \times 10 \times 4$ (M1)

$= 20$ (A1)

Use a straight-line model

4 marks **Grade C**

1. Find the gradient from two data points and form the equation.
2. Interpret the gradient and intercept in context.
3. Comment on when the model might not be valid.

Example. The length, L cm, of a spring is modelled as a linear function of the mass, m kg, hanging from it. When $m = 2$, $L = 15$ and when $m = 5$, $L = 21$.
(a) Find an equation for L in terms of m.
(b) Interpret the gradient in context.
(c) Give a reason why the model may not be valid when $m = 100$

Model answer.

(a) Gradient $\dfrac{21 - 15}{5 - 2} = 2$ (M1)

$L = 2m + 11$ (A1)

(b) The spring stretches by 2 cm for each extra kg. (B1)

(c) e.g. 100 kg is far outside the data, and the spring may pass its elastic limit or break. (B1)

Don’t lose marks

- Subtracting the coordinates in different orders on the top and bottom of the gradient fraction.
- Using $-m$ or $\dfrac{1}{m}$ for a perpendicular gradient instead of $-\dfrac{1}{m}$
- Leaving fractions when the question asks for integers a, b and c
- Writing $ax + by + c$ without '$= 0$', so it is not an equation.

More tips

Memory tricks

- Perpendicular gradient: flip and change the sign, $\frac{a}{b} \to -\frac{b}{a}$
- Sketch the points roughly: it catches sign errors and makes areas obvious.
- Check your line by substituting the given point(s) into your final equation.
- Gradient of $3x - 4y + 17 = 0$:

$$\begin{aligned} &-\frac{a}{b} \\ &= -\frac{3}{-4} \\ &= \frac{3}{4} \end{aligned}$$

Exam technique

- If no form is specified, any correct equation scores; if a form is specified (e.g. integers a, b, c), you must give it.
- Show the gradient calculation clearly; it is usually the first method mark.
- In multi-part questions, a quick sketch of the lines and points shows which lengths and heights to use.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Find the gradient between two points**

$m = \frac{y_2 - y_1}{x_2 - x_1}$: the change in y divided by the change in x

Grade E — **Find a line from a point and gradient**

Use $y - y_1 = m(x - x_1)$, then rearrange if a particular form is asked for.

Grade D — **Give a line's equation with integer coefficients**

Clear any fractions and collect all the terms on one side, with a, b and c integers.

Grade D — **Use the parallel and perpendicular gradient rules**

Parallel lines have equal gradients; perpendicular gradients multiply to give -1

Grade C — **Find midpoints, lengths and intersections**

Use the midpoint and distance formulae, and solve the equations of two lines simultaneously.

Grade C — **Use straight-line models in context**

Interpret the gradient as a rate of change and the intercept as a starting or fixed value.

Grade B — **Find the perpendicular bisector of a segment**

Use the midpoint of the segment and the negative reciprocal of its gradient.

Grade A — **Solve multi-step problems such as triangle areas**

Find vertices from intercepts and intersections, then use $\frac{1}{2} \times$ base $\times$ height.

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