Inequalities

P2.5 · Pure mathematics · revision notes

Solving linear and quadratic inequalities, including ones with brackets and fractions, and writing the solution with 'and', 'or' or set notation. You also show inequalities as regions on a graph. Quadratic inequalities often appear as the final step of a discriminant question.

Key facts

- Multiply or divide by a negative: reverse the inequality sign
- > 0: curve above the x-axis; < 0: curve below
- $\cup$ shape, roots $\alpha < \beta$: $> 0 \Rightarrow x < \alpha$ or $x > \beta$
- $\cup$ shape, roots $\alpha < \beta$: $< 0 \Rightarrow \alpha < x < \beta$
- 'or' / $\cup$ for two separate regions; 'and' / $\cap$ for an overlap
- $\{x : x > 3\}$ means 'the set of values of x such that $x > 3$'
- Boundary included for $\leq, \geq$ (solid); excluded for $<, >$ (dashed)
- Fraction inequality: multiply by the denominator squared

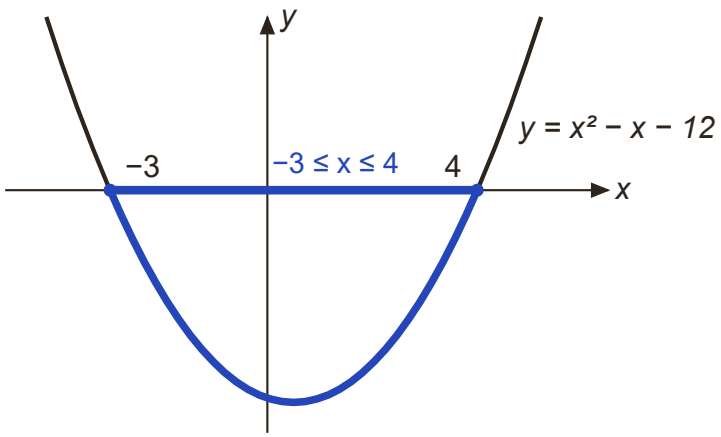
Notes

Linear inequalities

- Solve like an equation. If you multiply or divide both sides by a negative number, reverse the sign: $-3x < 12$ gives $x > -4$
- You can avoid this by collecting the x terms on the side where their coefficient is positive.
- With fractions, multiply every term by a positive common denominator: $\frac{x+1}{4} \leq \frac{2x-3}{3}$ gives $3x + 3 \leq 8x - 12$, so $x \geq 3$

Quadratic inequalities

- Rearrange so that one side is 0, then solve the equation to find the critical values.
- Sketch the curve. '> 0' means where the curve is above the x-axis; '< 0' means below.



x² − x − 12 ≤ 0 where the curve is on or below the x-axis, so −3 ≤ x ≤ 4.

- For a ∪-shaped curve with roots $\alpha < \beta$: > 0 gives $x < \alpha$ or $x > \beta$; < 0 gives $\alpha < x < \beta$
- Example: $x^2 - x - 12 \leq 0$ gives $(x - 4)(x + 3) \leq 0$, so $-3 \leq x \leq 4$
- Never divide both sides by an expression in x that could be negative or zero.

Writing the answer

- One interval: $-3 \leq x \leq 4$, or $\{x : -3 \leq x \leq 4\}$
- Two intervals: $x < -2$ or $x > 5$, or $\{x : x < -2\} \cup \{x : x > 5\}$. Never join them into one inequality.
- Values that satisfy both of two inequalities are the overlap, written with 'and' or ∩

Graphs and regions

- $y > x + 1$ is the region above the line $y = x + 1$; $y < x^2 - 4$ is the region below the curve.
- A boundary is included for $\leq$ or $\geq$ (usually drawn solid) and not included for $<$ or $>$ (usually drawn dashed).
- For $\dfrac{a}{x - b} > c$, multiply both sides by $(x - b)^2$, which is positive. Collect terms on one side, factorise and use a sketch. Remember $x \neq b$ Grade A or above only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find the set of values of x for a quadratic inequality

4 marks | Grade D

1. Rearrange to a quadratic compared with 0.
2. Solve the quadratic equation to find the critical values.
3. Sketch the parabola and choose the correct region.
4. Write the answer with 'or' or set notation.

Example. Find the set of values of x for which $2x^2 + 5x - 3 > 0$

Model answer.

$(2x - 1)(x + 3) = 0$ (M1)

Critical values $x = \frac{1}{2}$ and $x = -3$ (A1)

Choosing the outside region (M1)

$x < -3$ or $x > \frac{1}{2}$ (A1)

Find values that satisfy two inequalities

5 marks | Grade C

1. Solve each inequality separately.
2. Draw both solutions on a number line.
3. Write down the overlap.

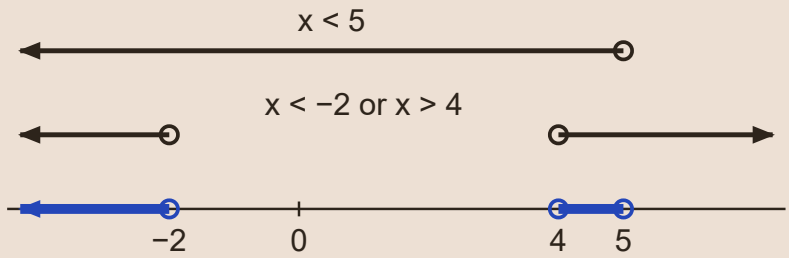
Example. Find the set of values of x for which both $3(x - 2) < x + 4$ and $x^2 - 2x - 8 > 0$

Model answer.

$3x - 6 < x + 4$, so $x < 5$ (B1)

$(x - 4)(x + 2) = 0$, critical values 4 and -2 (M1)

$x < -2$ or $x > 4$ (A1)



Combining with $x < 5$ (M1)

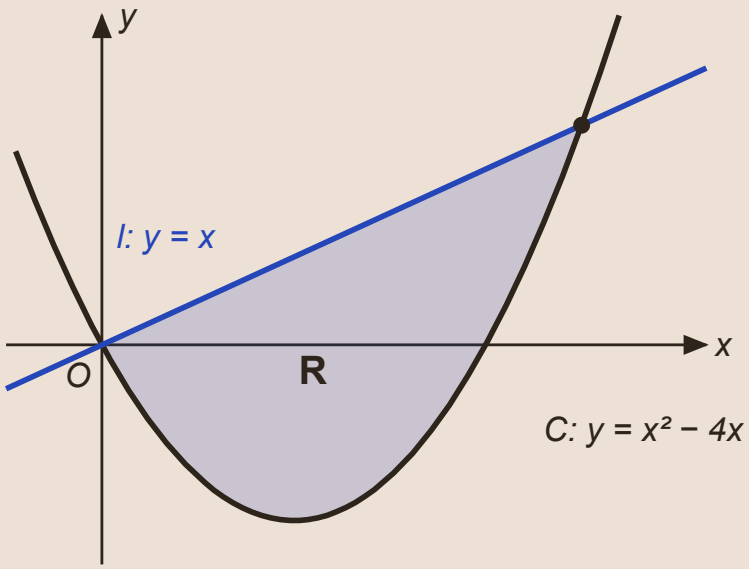
$\{x : x < -2\} \cup \{x : 4 < x < 5\}$ (A1)

Use inequalities to define a region

4 marks | Grade C

1. Find where the boundaries meet by solving their equations together.
2. Decide whether the region is above or below each boundary.
3. Use $\leq$, $\geq$ if the boundaries are included.

Example. The curve C has equation $y = x^2 - 4x$ and the line l has equation $y = x$. The finite region R is bounded by C and l, and includes its boundaries.
(a) Find the x-coordinates of the points where l meets C.
(b) Write down inequalities that define R.



Model answer.

(a) $x^2 - 4x = x$, so $x(x - 5) = 0$ (M1)

$x = 0$ and $x = 5$ (A1)

(b) $y \geq x^2 - 4x$ (B1)

$y \leq x$ (B1)

Solve an inequality with x in the denominator

5 marks | Grade A

1. Multiply both sides by the denominator squared.
2. Collect all terms on one side and factorise.
3. Find the critical values, including the value that makes the denominator 0.
4. Use a sketch to choose the region, excluding that value.

Example. Find the set of values of x for which $\frac{6}{x-1} < 2, x \neq 1$

Model answer.

$6(x-1) < 2(x-1)^2$	(M1)
$2(x-1)^2 - 6(x-1) > 0$, so $2(x-1)(x-4) > 0$	(A1)
Critical values $x = 1$ and $x = 4$	(B1)
Choosing the outside region	(M1)
$x < 1$ or $x > 4$	(A1)

Don't lose marks

- Writing two regions as one inequality, e.g. $\frac{1}{2} < x < -3$, or joining them with 'and'.
- Square rooting both sides: $x^2 > 9$ gives $x < -3$ or $x > 3$, not just $x > 3$
- Multiplying by $x - 1$, which might be negative, instead of $(x-1)^2$
- Stopping at the critical values without choosing the region.
- Forgetting to reverse the sign when dividing by a negative number.

More tips

Memory tricks

- Always sketch: a five-second parabola stops you choosing the wrong region.
- Test a value: pick a number in your answer and check it satisfies the original inequality.
- 'Find the set of values' is exam language for 'solve and give the answer as inequalities or a set'.
- For two inequalities, draw the solution lines one above the other on the same number line.

Exam technique

- Answer in the form asked for. For set notation use braces and $\cup$ or $\cap$ correctly.
- Show your sketch or sign diagram: it earns the method mark for choosing the region.
- Carry strict ($<$) or inclusive ($\leq$) signs through to the final answer exactly as the question needs.
- In discriminant questions, the final inequality in k is solved exactly like this.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E

Solve a linear inequality

Solve it like an equation, but reverse the sign if you multiply or divide by a negative number.

Grade D

Solve a quadratic inequality using a sketch

Find the critical values, sketch the parabola and choose the part above or below the x-axis.

Grade C

Write solutions using set notation

e.g. $\{x : x < -2\} \cup \{x : x > 5\}$ for two regions, $\{x : -2 < x < 5\}$ for one.

Grade C

Find values satisfying two inequalities together

Show both solution sets on a number line and find where they overlap.

Grade C

Define a region using inequalities

Decide which side of each line or curve the region is on, and whether the boundary is included.

Grade B

Solve inequalities using graphs

$f(x) > g(x)$ where the graph of f is above the graph of g; find the intersections first.

Grade A

Solve inequalities with x in the denominator

Multiply both sides by the square of the denominator, which is positive, then solve.

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