Simultaneous equations

P2.4 · Pure mathematics · revision notes

Solving two equations in two unknowns by elimination or substitution, including one linear and one quadratic equation. The solutions are the points where the graphs intersect, and substituting a line into a curve and using the discriminant tells you whether they meet at all. Typical questions are worth 4 to 7 marks.

Key facts

- Elimination: same signs subtract, different signs add
- Substitute back into the linear equation
- After substituting: $b^2 - 4ac > 0$ two points, $= 0$ tangent, < 0 no intersection
- One linear, one quadratic: use substitution
- Solutions = coordinates of the intersection points
- $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$

Notes

Linear simultaneous equations

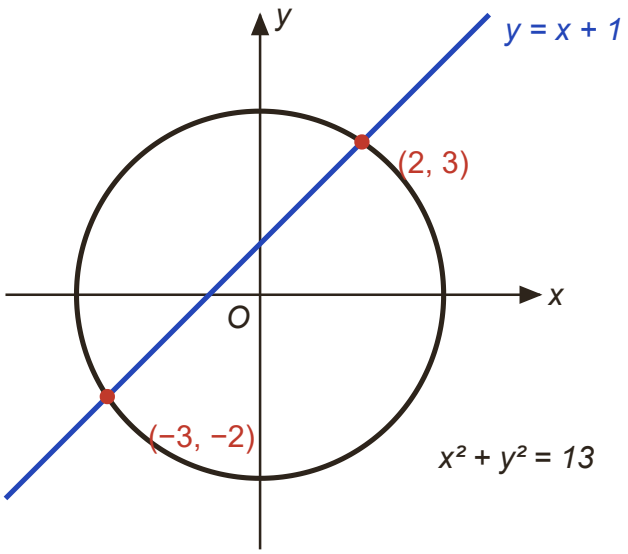
- Elimination: multiply one or both equations so that the coefficients of x (or y) are the same size; subtract if the signs are the same, add if they are different.
- Substitution: make x or y the subject of one equation and substitute it into the other.
- Check your answer in the equation you did not use last.

One linear and one quadratic

- Always use substitution. Rearrange the linear equation, choosing the variable that avoids fractions.
- Substitute into the quadratic, expand, collect every term on one side and solve the quadratic.
- Substitute each value back into the linear equation to find the matching value. Solutions come in pairs.
- Example: $y = x + 1$ and $x^2 + y^2 = 13$ give $x^2 + (x+1)^2 = 13$, so $2x^2 + 2x - 12 = 0$, i.e. $x^2 + x - 6 = 0$. So $x = 2, y = 3$ or $x = -3, y = -2$

What the solutions mean

- The solutions are the coordinates of the points where the two graphs intersect.



Each solution pair is a point where the line meets the curve: here (2, 3) and (−3, −2).

- After substituting a line into a curve you get a quadratic. Its discriminant tells you how they meet:
- $b^2 - 4ac > 0$: two points of intersection; $b^2 - 4ac = 0$: the line is a tangent (one point); $b^2 - 4ac < 0$: the line and curve do not meet.
- Curves with xy or y^2 terms work the same way; just expand every bracket carefully after substituting.

Grade A or above only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find constants from two conditions

3 marks | Grade D

1. Substitute each given point into the equation.
2. Solve the two linear equations simultaneously.
3. Check both points in your final equation.

Example. The curve $y = ax^2 + bx$ passes through the points $(1, 4)$ and $(2, 2)$. Find the values of a and b

Model answer.

$a + b = 4$ and $4a + 2b = 2$ (M1)

$2a + b = 1$, so subtracting: $a = -3$ (M1)

$a = -3$, $b = 7$ (A1)

Solve one linear and one quadratic equation

6 marks | Grade C

1. Rearrange the linear equation for one variable.
2. Substitute into the quadratic and simplify to a three-term quadratic equal to 0.
3. Solve it.
4. Substitute each value into the linear equation and pair the answers.

Example. In this question you must show all stages of your working.
Solve the simultaneous equations
$x + 2y = 5$
$x^2 + 2y^2 = 11$

Model answer.

$x = 5 - 2y$, so $(5 - 2y)^2 + 2y^2 = 11$ (M1)

$25 - 20y + 6y^2 = 11$, so $3y^2 - 10y + 7 = 0$ (A1)

$(3y - 7)(y - 1) = 0$ (M1)

$y = 1$ or $y = \frac{7}{3}$ (A1)

Substituting into $x = 5 - 2y$ (M1)

$x = 3, y = 1$ and $x = \frac{1}{3}, y = \frac{7}{3}$ (A1)

Show that a line and a curve do not meet

4 marks | Grade B

1. Substitute the line into the curve.
2. Simplify to a quadratic equal to 0.
3. Show that $b^2 - 4ac < 0$
4. Conclude in words.

Example. Show that the line $y = 3 - x$ does not meet the curve $x^2 + xy + 2y^2 = 1$

Model answer.

$x^2 + x(3 - x) + 2(3 - x)^2 = 1$ (M1)

$3x + 18 - 12x + 2x^2 = 1$, so $2x^2 - 9x + 17 = 0$ (A1)

$b^2 - 4ac = 81 - 136$ (M1)

$= -55 < 0$, so there are no real solutions and the line does not meet the curve. (A1)

Don't lose marks

- Squaring a bracket wrongly: $(5 - 2y)^2 = 25 - 20y + 4y^2$, not $25 + 4y^2$ or $25 - 4y^2$
- Substituting back into the quadratic, which can give extra wrong pairs; use the linear equation.

- Giving only the x values when the question asks for points or for both variables.
- Forgetting to multiply every term of an equation when using elimination.

More tips

Memory tricks

- Rearrange for the variable with coefficient 1 to avoid fractions.
- Keep answers paired: write '$x = 3, y = 1$' together, never two separate lists.
- Check each pair in BOTH original equations before moving on.

Exam technique

- Most of these questions say 'show all stages of your working': an equation solver's answers alone score little.
- Label your equations (1) and (2) and say what you do, e.g. 'substitute (1) into (2)'.
- If the question asks for coordinates, give them as (x, y) pairs.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Solve linear simultaneous equations by elimination**
Make the coefficients of one unknown match, then add or subtract the equations.

Grade D — **Find constants from two conditions**
Substitute two given points into a formula to get two linear equations, then solve them.

Grade D — **Solve one linear and one quadratic equation**
Rearrange the linear equation for x or y and substitute into the quadratic.

Grade C — **Find the coordinates of intersection points**
Solve for one variable, then substitute each value into the linear equation to get the pairs.

Grade C — **Interpret solutions as intersections of graphs**
Two solutions: the line crosses the curve twice; one: it is a tangent; none: they do not meet.

Grade B — **Use the discriminant to decide if graphs meet**
Substitute the line into the curve, form a quadratic and test the sign of $b^2 - 4ac$

Grade A — **Handle curves with xy and y-squared terms**
Substitute carefully, expanding brackets like $(3 - x)^2$ and $x(3 - x)$ in full.

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