Surds

P2.2 · Pure mathematics · revision notes

Surds are exact roots such as $\sqrt{12}$ that are not whole numbers. You need to simplify them, multiply out brackets containing them and rationalise denominators. They are tested in short 'show all stages of your working' questions and are expected for exact answers in coordinate geometry, trigonometry and calculus.

Key facts

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- $\sqrt{a} \times \sqrt{a} = a$
- $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$
- $\dfrac{k}{\sqrt{a}} = \dfrac{k\sqrt{a}}{a}$
- $\dfrac{k}{a + \sqrt{b}} = \dfrac{k(a - \sqrt{b})}{a^2 - b}$
- Square numbers to look for: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

Notes

Simplifying surds

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- To simplify, find the largest square factor: $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$
- $\sqrt{a} \times \sqrt{a} = a$, so $(2\sqrt{5})^2 = 4 \times 5 = 20$
- $\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$: for example $\sqrt{9 + 16} = 5$, not 7.
- Only like surds can be collected: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ does not simplify.

Expanding brackets

- Expand as in algebra, using $\sqrt{a} \times \sqrt{a} = a$:

$$\begin{aligned}(\sqrt{7} - 2)^2 &= 7 - 4\sqrt{7} + 4 \\ &= 11 - 4\sqrt{7}\end{aligned}$$

- Difference of two squares: $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$, which has no surd. This is the key to rationalising.

Rationalising the denominator

- Rationalising means rewriting a fraction so that there is no surd in the denominator.
- For $\frac{k}{\sqrt{a}}$, multiply top and bottom by $\sqrt{a}$:

$$\begin{aligned}&\frac{10}{\sqrt{5}}\\&=\frac{10\sqrt{5}}{5}\\&=2\sqrt{5}\end{aligned}$$

- For $\frac{k}{a-\sqrt{b}}$, multiply top and bottom by the conjugate $a+\sqrt{b}$:

$$\begin{aligned}&\frac{4}{3-\sqrt{5}}\\&=\frac{4(3+\sqrt{5})}{9-5}\\&=3+\sqrt{5}\end{aligned}$$

- For $\frac{k}{\sqrt{a}+\sqrt{b}}$, multiply by $\sqrt{a}-\sqrt{b}$: the denominator becomes $a-b$
- Whatever you multiply the bottom by, you must multiply the top by the same thing.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Simplify surds into the form $k\sqrt{b}$ 2 marks **Grade E**

1. Simplify each surd using its largest square factor.
2. Collect the like surds.

Example. Express $\sqrt{75}-\sqrt{12}+\sqrt{27}$ in the form $k\sqrt{3}$, where k is an integer.

Model answer.

$\sqrt{75}=5\sqrt{3}$, $\sqrt{12}=2\sqrt{3}$, $\sqrt{27}=3\sqrt{3}$ (at least two correct) (M1)

$5\sqrt{3}-2\sqrt{3}+3\sqrt{3}=6\sqrt{3}$ (A1)

Rationalise the denominator

3 marks | Grade C

1. Multiply the top and bottom by the conjugate of the denominator.
2. Simplify the denominator using $(a-b)(a+b)=a^2-b^2$
3. Simplify fully and give the answer in the form asked for.

Example. In this question you must show all stages of your working.

Express $\dfrac{6}{\sqrt{7}-2}$ in the form $a+b\sqrt{7}$, where a and b are integers.

Model answer.

$\dfrac{6}{\sqrt{7}-2}\times\dfrac{\sqrt{7}+2}{\sqrt{7}+2}$ (M1)

Denominator $7-4=3$ (A1)

$\dfrac{6(\sqrt{7}+2)}{3}=4+2\sqrt{7}$ (A1)

Simplify a fraction with surds on top and bottom

4 marks | Grade B

1. Multiply top and bottom by the conjugate of the denominator.
2. Expand the numerator fully (four terms).
3. Simplify the denominator, then collect terms.

Example. In this question you must show all stages of your working.

Show that $\dfrac{5+\sqrt{3}}{2+\sqrt{3}}$ can be written as $a+b\sqrt{3}$, where a and b are integers to be found.

Model answer.

$\dfrac{(5+\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$ (M1)

Numerator $10-5\sqrt{3}+2\sqrt{3}-3$ (M1)

Denominator $4-3=1$ (A1)

$7-3\sqrt{3}$, so $a=7$, $b=-3$ (A1)

Solve an equation, giving an exact answer

4 marks | **Grade A**

1. Collect the x terms on one side and factorise out x
2. Divide, then rationalise the denominator.
3. Simplify to the form asked for.

Example. Solve $3x + 4 = x\sqrt{5} + 2\sqrt{5}$, giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

Model answer.

$x(3 - \sqrt{5}) = 2\sqrt{5} - 4$ (M1)

$x = \dfrac{(2\sqrt{5} - 4)(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})}$ (M1)

$= \dfrac{6\sqrt{5} + 10 - 12 - 4\sqrt{5}}{4}$

$= \dfrac{2\sqrt{5} - 2}{4}$ (A1)

$x = -\dfrac{1}{2} + \dfrac{1}{2}\sqrt{5}$ (A1)

Don't lose marks

- Multiplying only the denominator by the conjugate: the top must be multiplied too.
- Writing $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$
- Sign slips when expanding: $-\sqrt{3} \times -\sqrt{3} = +3$ and $(2 - \sqrt{3})(2 + \sqrt{3}) = 1$
- Not simplifying fully, e.g. leaving $\dfrac{6\sqrt{2}}{2}$ instead of $3\sqrt{2}$, or $\sqrt{12}$ in a final answer.

More tips

Memory tricks

- Conjugate: change the sign in the middle, e.g. $3 - \sqrt{5}$ becomes $3 + \sqrt{5}$
- Check with your calculator: your simplified answer must have the same decimal value as the original.
- To simplify $\sqrt{n}$, try dividing n by 4, 9, 16, 25, 36, ... to find the largest square factor.
- Simplify surds before you do anything else: $\sqrt{50} = 5\sqrt{2}$ makes later steps easier.

Exam technique

- When the question says 'show all stages of your working', write the multiplication by the conjugate and the expanded numerator. A calculator answer alone scores nothing.
- Give the answer in exactly the form asked, e.g. $a + b\sqrt{3}$, and state a and b if asked.
- Exact answers in other topics (lengths, coordinates, areas) usually need simplified surds.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E **Simplify a surd using a square factor**

$$\begin{aligned}\sqrt{72} &= \sqrt{36}\sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

Grade E **Add and subtract like surds**

$$\begin{aligned}\sqrt{50} + \sqrt{8} &= 5\sqrt{2} + 2\sqrt{2} \\ &= 7\sqrt{2}\end{aligned}$$

Grade D **Expand brackets containing surds**

Multiply out, then collect like terms: $(3+\sqrt{5})(2-\sqrt{5}) = 1 - \sqrt{5}$

Grade C **Rationalise a single-surd denominator**

Multiply top and bottom by the surd:

$$\begin{aligned}&\frac{6}{\sqrt{3}} \\ &= \frac{6\sqrt{3}}{3} \\ &= 2\sqrt{3}\end{aligned}$$

Grade B **Rationalise denominators using the conjugate**

For $a + \sqrt{b}$, multiply top and bottom by $a - \sqrt{b}$; the denominator becomes $a^2 - b$

Grade A **Solve linear equations with surd coefficients**

Collect the x terms, factorise, divide and rationalise to give an exact answer.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.