Modelling with functions

P2.11 · Pure mathematics · revision notes

Using functions (linear, quadratic, reciprocal and others) to model real situations: finding constants from data, making predictions, interpreting features in context, and commenting on the limitations of a model and how to refine it. These are multi-part questions that mix algebra with short 'interpret' or 'criticise' parts worth 1 or 2 marks.

Key facts

- Initial value: substitute $t = 0$
- Roots: when the quantity is zero (e.g. hits the ground)
- Extrapolation (outside the data) is unreliable
- Always answer in context, with units
- Maximum of $q - b(t - p)^2$ ($b > 0$) is q, when $t = p$
- Linear model $y = mx + c$: m = rate of change, c = starting or fixed value
- Limitations: negative values, unlimited growth, ignored factors

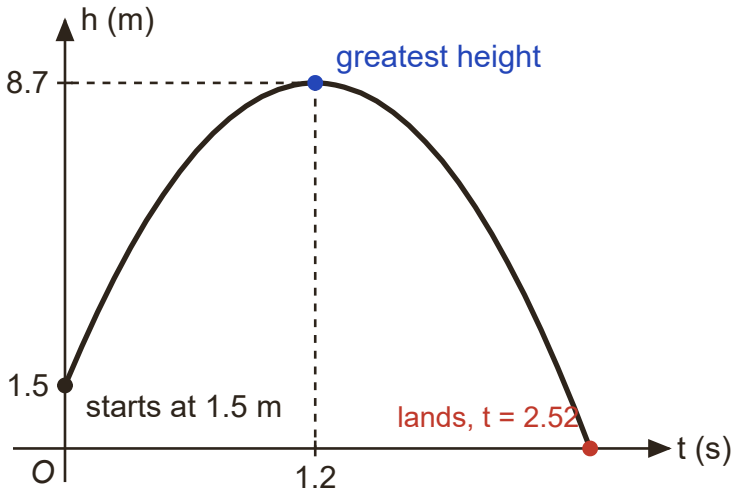
Notes

Using a model

- A mathematical model describes a real situation with a function, e.g. the height h m of a ball t seconds after it is thrown: $h = 1.5 + 12t - 5t^2$
- Substitute to make predictions, and always give answers in context with units.
- Find unknown constants by substituting given data (such as the initial value when $t = 0$) and solving the equations.

Interpreting features

- The value when $t = 0$ is the initial value: here the ball is thrown from 1.5 m.
- The turning point of a quadratic model gives a maximum or minimum. Completing the square: $h = 8.7 - 5(t - 1.2)^2$, so the greatest height is 8.7 m, after 1.2 s.



The intercept, the turning point and the root each mean something in context.

- Roots often give start or end times, e.g. when the ball hits the ground ($h = 0$). Reject negative times.
- In a linear model $y = mx + c$, m is a rate of change (e.g. cost per item) and c is a fixed or starting value.

Limitations and refinements

- A model is only valid over a sensible domain: time cannot be negative, and the height model stops applying when the ball lands.
- Predictions far outside the range of the data (extrapolation) are unreliable.
- Criticise in context, e.g. 'the model predicts the number of visitors becomes negative after 14 months, which is impossible'.
- Refinements: include a factor that was ignored (air resistance, wind, a maximum capacity), change the type of function, or restrict the domain.
- If real data are given, compare them with the model's predictions: close agreement supports the model.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, ft = follow through: you still earn the mark if you use your own earlier wrong answer correctly.

Use a quadratic model: maximum and landing time

6 marks | Grade B

1. Put $t = 0$ for the starting value.
2. Complete the square to find the maximum and when it happens.
3. Set the model equal to 0 to find when it lands, rejecting negative times.
4. Give each answer in context with units.

Example. A ball is thrown upwards from a platform. Its height above the ground, h metres, t seconds after it is thrown is modelled by $h = 2 + 8t - 4t^2$.
(a) State the height of the platform.
(b) Write h in the form $a - b(t - c)^2$.
(c) Hence state the maximum height of the ball and the time at which it occurs.
(d) Find the time at which the ball hits the ground.

Model answer.

(a) 2 m (B1)

(b) $h = -4(t^2 - 2t) + 2$
$= -4(t - 1)^2 + 4 + 2$ (M1)

$h = 6 - 4(t - 1)^2$ (A1)

(c) 6 m, after 1 second (B1 ft)

(d) $4(t - 1)^2 = 6$, so $t = 1 + \sqrt{1.5}$ (M1)

$t = 2.22$ seconds (3 s.f.); $t = 1 - \sqrt{1.5}$ is negative so is rejected (A1)

Find and interpret constants in a model

4 marks | Grade D

1. Substitute each data pair into the model.
2. Solve the equations simultaneously.
3. Interpret each constant in words, in context.

Example. The cost, £C, of hiring a van for d days is modelled by $C = a + bd$. Hiring the van for 3 days costs £165 and hiring it for 7 days costs £305.
(a) Find the values of a and b.
(b) Interpret the value of a in this context.

Model answer.

(a) $a + 3b = 165$ and $a + 7b = 305$, solved simultaneously (M1)

$b = 35$ (A1)

$a = 60$ (A1)

(b) A fixed charge of £60 for each hire, whatever the number of days (B1)

Evaluate a model's prediction

2 marks | Grade C

1. Substitute the value into the model.
2. Decide whether the result is realistic in context.
3. Give a reason that refers to the context.

Example. The number of visitors, V thousand, to a new museum t months after it opens is modelled by $V = 12 + 6t - \frac{1}{2}t^2$.
(a) Use the model to find V when $t = 14$.
(b) Comment on your answer.

Model answer.

(a) $V = 12 + 84 - 98 = -2$ (B1)

(b) A negative number of visitors is impossible, so the model is not valid for $t = 14$. (B1)

Find an optimum and give a limitation

4 marks | Grade B

1. Complete the square (or use $x = -\frac{b}{2a}$) to find the maximum.
2. Interpret the values in context, remembering any units such as thousands.
3. Give a limitation that refers to this situation.

Example. A company models its monthly profit, £P thousand, from selling a product at £x each by $P = -2x^2 + 36x - 100$.
(a) Find the selling price that gives the maximum profit, and the maximum profit.
(b) Give one limitation of the model.

Model answer.

(a) $P = 62 - 2(x - 9)^2$ (M1)

Price £9 (A1)

Maximum profit £62 000 (A1)

(b) e.g. the model ignores changes in costs or in competitors' prices, so it may not hold for prices far from those it was based on (B1)

Don’t lose marks

- Giving a vague limitation such as 'it might not be accurate' with no reason linked to the context.
- Forgetting units or scales, e.g. P is in thousands of pounds.
- Keeping a negative time when solving a quadratic instead of rejecting it.
- Rounding too early when finding constants; keep exact values until the end.

More tips

Memory tricks

- Context check: is your answer sensible (not a negative time, not a 40 m child)?
- For 'criticise' questions, try an extreme value in the model and see whether the result is realistic.
- Limitations say what the model ignores or where it fails; refinements add or change something to fix that.

Exam technique

- 1-mark 'comment' or 'evaluate' parts need a reason linked to the context, not just 'the model is wrong'.
- When asked to evaluate a model against data, quote the model's value and the real value and compare them.
- Give final answers to a sensible accuracy (usually 3 significant figures) with units, and say what they mean.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Use a model to make a prediction**
Substitute into the formula and give the answer in context, with units.

Grade D — **Find constants in a model from data**
Substitute given pairs of values to form equations and solve for the unknown constants.

Grade C — **Interpret constants and features in context**
e.g. the value at $t = 0$ is the starting value; the vertex of a quadratic is a greatest height.

Grade C — **State a sensible domain for a model**
Only values that make sense, e.g. from $t = 0$ until the object hits the ground.

Grade B — **Find a maximum using completed square form**
Write the model as $a - b(t - c)^2$ to read off the maximum a when $t = c$

Grade B — **Criticise the limitations of a model**
Give a reason in context, e.g. it predicts negative values or unlimited growth, or relies on extrapolation.

Grade A — **Suggest and justify refinements to a model**
e.g. include air resistance, or use a function that levels off instead of growing forever.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.