Moments, beams and rods in equilibrium

M9.1a · Mechanics · revision notes

A moment is the turning effect of a force about a point. A rigid body such as a beam or rod is in equilibrium only if the resultant force is zero and the resultant moment about every point is zero. Questions involve beams on supports, rods hanging from strings, non-uniform rods and hinged rods held by strings at an angle.

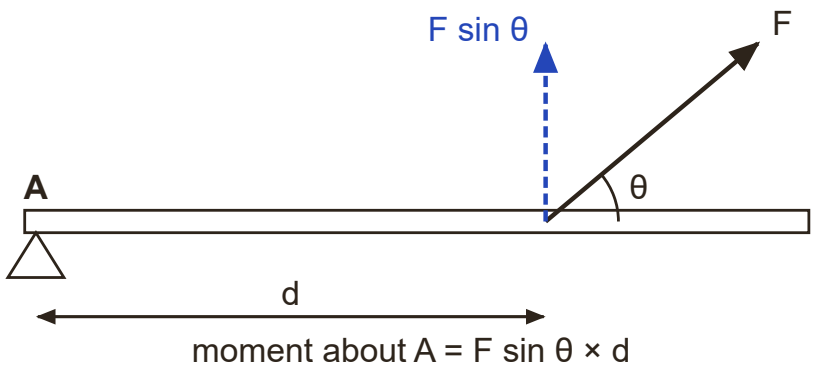
Key facts

- Moment = force × perpendicular distance (N m)
- Rigid body in equilibrium: resultant force = 0 and resultant moment = 0
- Uniform rod: weight at the midpoint; light rod: no weight
- Take moments about a point where an unknown force acts
- Force at θ to a rod, at distance d: moment $= Fd\sin\theta$
- In equilibrium: clockwise moments = anticlockwise moments, about any point
- Hinge force: components H, V; magnitude $\sqrt{H^2+V^2}$

Notes

Moments

- The **moment** of a force about a point is its turning effect: force × perpendicular distance from the point to the line of action of the force. Units: N m.
- Always state the sense: clockwise or anticlockwise.
- A force whose line of action passes through the point has zero moment about that point.
- For a force F acting at distance d along a rod, at angle θ to the rod, the moment about the point is $Fd\sin\theta$: only the component perpendicular to the rod turns it.



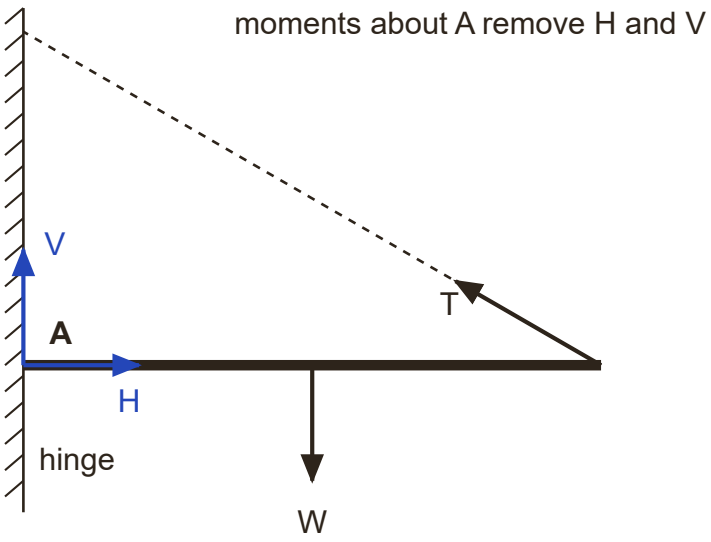
Only the component perpendicular to the rod has a moment

Rigid bodies in equilibrium

- A rigid body (a rod, beam or plank) is in equilibrium when the resultant force is zero **and** the resultant moment about any point is zero.
- So you can resolve (vertically and horizontally) and take moments: in equilibrium, total clockwise moment = total anticlockwise moment about any point.
- Take moments about the point where an unknown force you don't want acts: it then has no moment.
- A **uniform** rod's weight acts at its midpoint. A **non-uniform** rod's weight acts at its centre of mass, often at an unknown distance x from one end. A **light** rod has no weight.
- Loads modelled as particles act at a single point on the rod.

Supports, strings and hinges

- A support pushes up on a beam with a normal reaction; a vertical string pulls up with a tension.
- A hinge can exert a force in any direction. Give it horizontal and vertical components H and V; its magnitude is $\sqrt{H^2 + V^2}$



Give a hinge force two components, H and V

- If a component comes out negative, it acts in the opposite direction to the one you drew.
- Check your answers by taking moments about a second point.

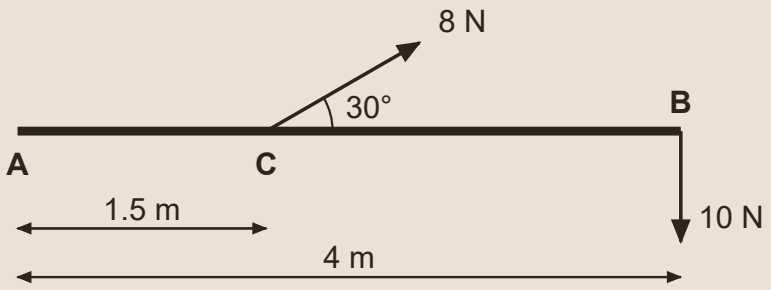
How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find the resultant moment about a point 3 to 4 marks **Grade E**

1. For each force, find the perpendicular distance from the point (or the perpendicular component of the force).
2. Multiply to get each moment and note its sense.
3. Resultant moment = clockwise total − anticlockwise total; state the sense.

Example. A light rod AB of length 4 m is horizontal, with A to the left of B. A force of 10 N acts vertically downwards at B. A force of 8 N acts at the point C of the rod, where $AC = 1.5$ m; it acts upwards at an angle of 30° to the rod. Find the magnitude and sense of the resultant moment of these two forces about A.



Model answer.

Moment of the 10 N force about A: $10 \times 4 = 40$ N m clockwise (B1)

Moment of the 8 N force about A: $8 \sin 30° \times 1.5 = 6$ N m anticlockwise (M1 A1)

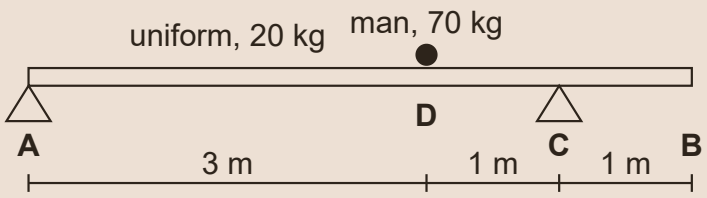
Resultant moment $= 40 - 6 = 34$ N m clockwise (A1)

Beam resting on two supports

4 to 6 marks **Grade D**

1. Draw the beam with every force: the reactions at the supports, the beam's weight (at its midpoint if uniform) and any loads.
2. Take moments about one support to find the reaction at the other.
3. Resolve vertically to find the remaining reaction, or take moments about the other support as a check.

Example. A uniform plank AB, of length 5 m and mass 20 kg, rests horizontally on two supports, one at A and one at C, where $AC = 4$ m. A man of mass 70 kg stands on the plank at D, where $AD = 3$ m. The man is modelled as a particle. Find the magnitude of the reaction at each support.



Model answer.

Moments about A: $R_C \times 4 = 20g \times 2.5 + 70g \times 3$ (M1 A1)

$R_C = 65g = 637$ N (A1)

Vertically: $R_A + R_C = 90g$ (M1), so $R_A = 25g = 245$ N (A1)

Non-uniform rod held by vertical strings

5 to 6 marks **Grade C**

1. Let the centre of mass be x m from one end.
2. Resolve vertically to link the tensions and the weight.
3. Take moments about one end (or about one string) to find x

Example. A non-uniform rod AB, of length 3 m and mass 12 kg, hangs in equilibrium in a horizontal position from two light vertical strings. One string is attached at A and the other at the point C, where $CB = 0.5$ m. The tension in the string at C is twice the tension in the string at A. Find the distance of the centre of mass of the rod from A

Model answer.

Let the tension at A be T: $T + 2T = 12g$ (M1), so $T = 4g$ (A1)

Moments about A: $2T \times 2.5 = 12g \times x$ (M1 A1)

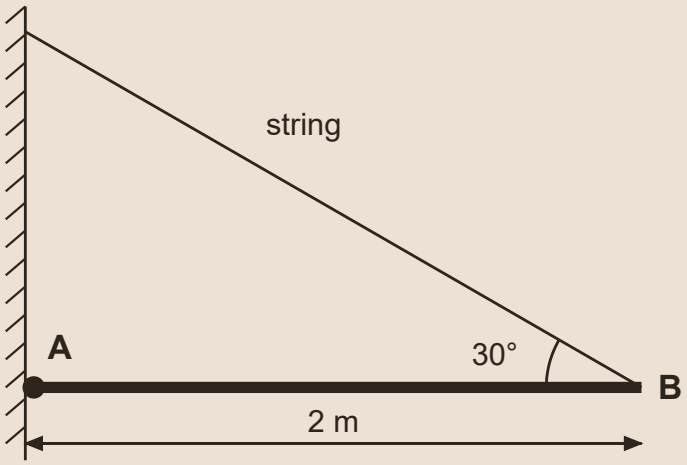
$8g \times 2.5 = 12gx$, so $x = \frac{5}{3}$ m, about 1.67 m (A1)

Hinged rod held by a string at an angle

7 to 10 marks **Grade A**

1. Take moments about the hinge to find the tension: only the component of the tension perpendicular to the rod has a moment.
2. Resolve horizontally and vertically to find the components H and V of the hinge force.
3. Combine: magnitude $\sqrt{H^2 + V^2}$, direction from $\tan^{-1}\dfrac{V}{H}$

Example. A uniform rod AB, of length 2 m and mass 5 kg, is freely hinged at A to a vertical wall. The rod is held in a horizontal position by a light string attached to B and to a point on the wall vertically above A. The string makes an angle of 30° with the rod.
(a) Find the tension in the string.
(b) Find the magnitude of the force exerted on the rod by the hinge.



Model answer.

(a) Moments about A: $T\sin 30° \times 2 = 5g \times 1$ (M1 A1), $T = 5g = 49$ N (A1)

(b) Horizontally: $H = T\cos 30°$
$= 42.43...$ N (M1 A1)

Vertically: $V + T\sin 30° = 5g$, so $V = 24.5$ N (M1 A1)

Magnitude $= \sqrt{42.43^2 + 24.5^2}$ (M1) = 49 N (A1)

Don’t lose marks

- Using the distance along the rod when the force is not perpendicular to it.
- Forgetting the weight of the rod, or giving a light rod a weight.
- Only taking moments: you usually need to resolve as well to find every force.
- Mixing moments about different points in one equation.
- Giving only one component when asked for the magnitude of a hinge force.

More tips

Memory tricks

- Take moments about the point that has the most unknown forces acting through it.
- Check reactions: when all the forces are vertical, the reactions add up to the total weight.
- Keep answers as multiples of g (such as $65g$) in your working; it makes checking easier.
- Perpendicular distance for a force at angle θ to the rod: $d\sin\theta$

Exam technique

- Write 'M(A):' to show which point you are taking moments about.
- Explain 'uniform' as 'the weight acts at the midpoint' and 'light' as 'its weight can be ignored'.
- When asked for a force, give its magnitude and, if needed, its direction.
- Give numerical answers found using $g = 9.8$ to 2 or 3 significant figures.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Calculate the moment of a force**
Moment = force × perpendicular distance from the point, stating clockwise or anticlockwise.

Grade D — **Find the reactions on a beam on two supports**
Take moments about one support, then resolve vertically for the other reaction.

Grade C — **State the conditions for equilibrium of a rigid body**
The resultant force is zero and the resultant moment about any point is zero.

Grade C — **Find the centre of mass of a non-uniform rod**
Put the weight at an unknown distance x from one end and take moments.

Grade B — **Find the moment of a force at an angle**
Use the perpendicular component $F\sin\theta$, or the perpendicular distance $d\sin\theta$

Grade A — **Solve hinged rod problems with non-parallel forces**
Take moments about the hinge, then resolve to find the components of the force at the hinge.

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