Friction

M8.6 · Mechanics · revision notes

Friction is modelled by $F \leqslant \mu R$: it takes whatever value is needed to stop sliding, up to the limiting value μR, which it reaches when the object is moving or on the point of moving. Questions involve rough horizontal surfaces and rough slopes, forces at angles, and deciding whether an object moves.

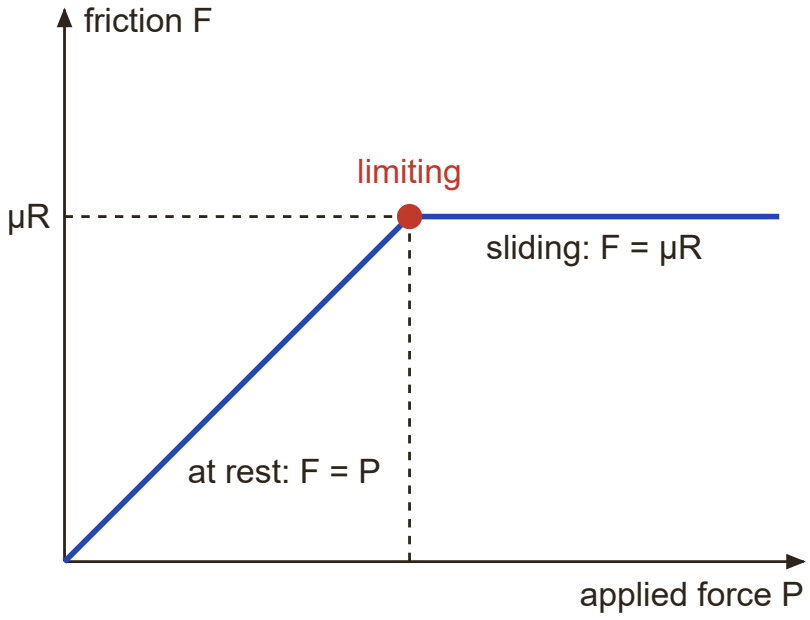
Key facts

- $F \leqslant \mu R$
- Friction opposes the motion (or the tendency to move)
- Sliding down: $a = g(\sin\theta - \mu\cos\theta)$
- On the point of slipping under its weight alone: $\mu = \tan\theta$
- Moving, or in limiting equilibrium: $F = \mu R$
- Rough slope, no other perpendicular forces: $R = mg\cos\theta$
- Moving up with no other force: deceleration $g(\sin\theta + \mu\cos\theta)$
- μ has no units

Notes

The friction model

- Friction acts along the surface of contact, opposing the motion, or the motion that would happen if there were no friction.
- $F \leqslant \mu R$, where R is the normal reaction and μ is the **coefficient of friction**, a number with no units ($\mu \geqslant 0$). A smooth surface has $\mu = 0$
- When an object is at rest, friction takes only the value needed to stop it sliding, up to a maximum of μR
- Friction is **limiting**, $F = \mu R$, when the object is sliding or is in **limiting equilibrium** (on the point of sliding).



Friction matches the push up to μR; then the object slides and F = μR

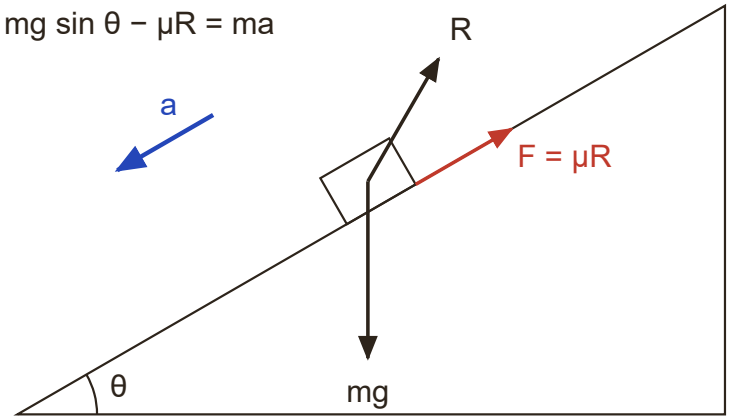
- In this model friction does not depend on the speed or on the area of contact.

Method

- Find R first by resolving perpendicular to the surface. Any force with a component perpendicular to the surface changes R
- Then, parallel to the surface: if the object is moving, $F = \mu R$ and use $F = ma$; if it is at rest, use equilibrium.
- To decide whether an object moves, find the friction needed to keep it at rest and compare it with μR. If more than μR is needed, the object slides.

Rough slopes

- With no other forces perpendicular to the slope, $R = mg\cos\theta$
- Sliding down: $mg\sin\theta - \mu R = ma$, so $a = g(\sin\theta - \mu\cos\theta)$



Sliding down: friction acts up the slope

- Moving up the slope (e.g. projected up it), friction acts down the slope and the deceleration is $g(\sin\theta + \mu\cos\theta)$
- A particle resting on a slope under its weight alone stays at rest if $\mu \geqslant \tan\theta$, and is on the point of slipping when $\mu = \tan\theta$
- After a particle projected up a slope comes to rest, it slides back down only if $\tan\theta > \mu$
- For the range of a force that keeps a particle at rest, use two limiting cases: about to move up (friction down the slope) and about to move down (friction up the slope).

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, dM = depends on the earlier method mark.

Rough horizontal surface: find the acceleration 3 to 5 marks **Grade D**

1. Resolve vertically to find R
2. Friction $= \mu R$, because the object is moving.
3. Apply $F = ma$ horizontally.

Example. A box of mass 5 kg is pulled along rough horizontal ground by a horizontal force of magnitude 30 N. The coefficient of friction between the box and the ground is 0.4. Find the acceleration of the box.

Model answer.

$R = 5g = 49$ N (B1)

$F = 0.4 \times 49 = 19.6$ N (M1)

$30 - 19.6 = 5a$ (M1), $a = 2.1$ m s^{-2} (2 s.f.) or 2.08 m s^{-2} (3 s.f.) (A1)

Force at an angle with friction

5 to 7 marks **Grade C**

1. Resolve vertically: $R = mg - T\sin\theta$ for a pull above the horizontal.
2. Friction $= \mu R$ if the object is moving or about to move.
3. Resolve horizontally: constant speed means the forces balance.
4. Solve for the unknown, remembering that T appears in both equations.

Example. A sledge of mass 12 kg is pulled at constant speed across rough horizontal ground by a rope inclined at 30° above the horizontal. The coefficient of friction between the sledge and the ground is 0.25. Find the tension in the rope.

Model answer.

Vertically: $R + T\sin 30° = 12g$, so $R = 117.6 - 0.5T$ (M1 A1)

Horizontally: $T\cos 30° = 0.25R$ (M1 A1)

$T\cos 30° = 0.25(117.6 - 0.5T)$, so $T(\cos 30° + 0.125) = 29.4$ (dM1)

$T = 30$ N (2 s.f.) or 29.7 N (3 s.f.) (A1)

Projected up a rough slope: does it slide back?

7 to 10 marks **Grade A**

1. Resolve perpendicular to the slope: $R = mg\cos\alpha$
2. Going up, friction and $mg\sin\alpha$ both act down the slope: find the deceleration.
3. Use suvat for the distance travelled before it stops.
4. At rest, compare $mg\sin\alpha$ with the limiting friction $\mu mg\cos\alpha$. If the particle slides back, friction now acts up the slope.

Example. A particle is projected with speed 8 m s^{-1} up a line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \dfrac{3}{4}$. The coefficient of friction between the particle and the plane is 0.5.
(a) Find the deceleration of the particle as it moves up the plane.
(b) Find the distance it moves up the plane before coming to rest.
(c) Determine whether the particle then slides back down the plane, giving a reason.

Model answer.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, so $R = 0.8mg$ (B1)

(a) $-0.6mg - 0.5(0.8mg) = ma$ (M1 A1), so $a = -g$ and the deceleration is 9.8 m s^{-2} (A1)

(b) $0 = 8^2 - 2(9.8)s$ (M1), $s = 3.3$ m (2 s.f.) or 3.27 m (3 s.f.) (A1)

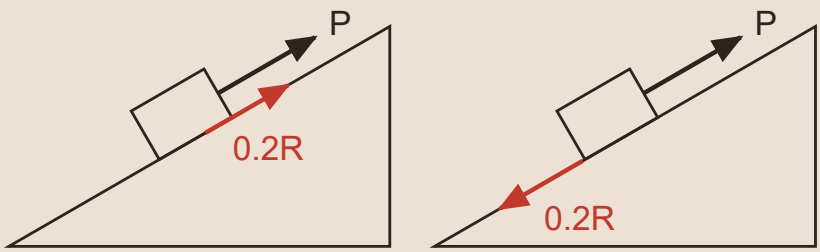
(c) Component of weight down the slope $= 0.6mg$; greatest possible friction $= 0.5 \times 0.8mg = 0.4mg$ (M1). Since $0.6mg > 0.4mg$, the particle slides back down (A1)

Limiting equilibrium: range of values of a force 5 to 7 marks **Grade A***

1. Find R (here $mg\cos\theta$, as the applied force is parallel to the slope).
2. About to slide down: friction μR acts up the slope, giving the least force.
3. About to slide up: friction μR acts down the slope, giving the greatest force.
4. Write the answer as an inequality.

Example. A particle of mass 2 kg rests on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is 0.2. The particle is held in equilibrium by a force of magnitude P newtons acting up the line of greatest slope. Find the range of possible values of P

Model answer.



$R = 2g\cos 30°$
$= 16.97...$ N, so $\mu R = 3.394...$ N (M1)

About to slide down: $P + 0.2R = 2g\sin 30°$ (M1 A1), $P = 6.405...$

About to slide up: $P = 2g\sin 30° + 0.2R$ (M1 A1), $P = 13.19...$

$6.41 \leqslant P \leqslant 13.2$ (3 s.f.) (A1)

Don't lose marks

- Writing $F = \mu R$ for an object at rest that is not on the point of moving.
- Using $R = mg$ on a slope, or when a force at an angle acts.
- Putting friction in the wrong direction, especially for motion up a slope.
- Forgetting that friction reverses direction when the particle reverses.
- Giving μ units, or a negative value.

More tips

Memory tricks

- On a slope: moving up → friction acts down; moving down → friction acts up.
- Quick test for a particle on a slope under its weight alone: compare $\tan\theta$ with μ
- With $\tan\alpha = \frac{3}{4}$, use $\sin\alpha = 0.6$ and $\cos\alpha = 0.8$; the mass m often cancels.
- Sense check: the friction on a body at rest can never be more than μR

Exam technique

- Justify $F = \mu R$ in words: 'friction is limiting because the particle is moving' (or 'about to move').
- Mark the direction of friction on your diagram before writing any equations.
- In 'show that $\mu = \ldots$' questions, show every step and reach the exact value given.
- For a range of values, give a double inequality such as $6.41 \leqslant P \leqslant 13.2$

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Know the direction in which friction acts**
Along the surface, opposing the motion, or the motion that would happen without friction.

Grade D — **Use $F = \mu R$ on rough level ground**
Find R by resolving vertically; when the object slides, the friction is μR

Grade C — **Understand when friction is limiting**
Friction equals μR only when the object is moving or about to move; otherwise $F \leqslant \mu R$

Grade C — **Include an angled force with friction**
A force with a vertical component changes R, and so changes the limiting friction μR

Grade B — **Solve rough inclined plane problems**
Resolve perpendicular to the slope for R, then use $F = ma$ or equilibrium along the slope.

Grade A — **Decide whether a particle moves**
Compare the friction needed for equilibrium with the greatest friction available, μR

Grade A* — **Find the range of forces for equilibrium**
Use both limiting cases: about to slide up (friction down the slope) and about to slide down (friction up the slope).

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