Resolving forces and inclined planes

M8.2, M8.4, M8.5 · Mechanics · revision notes

Resolving forces into two perpendicular components lets you deal with forces at angles and with particles on inclined planes. You resolve along and perpendicular to the direction of motion (along and perpendicular to a slope), then use equilibrium or $F = ma$

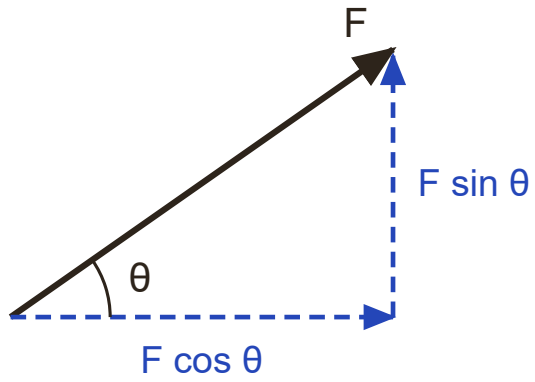
Key facts

- Components of F: $F\cos\theta$ along, $F\sin\theta$ perpendicular
- Smooth slope, no other forces: $a = g\sin\theta$, $R = mg\cos\theta$
- Horizontal force P on a slope: $P\cos\theta$ along, $P\sin\theta$ perpendicular
- No acceleration perpendicular to the motion: the forces balance
- Slope at θ: $mg\sin\theta$ down the slope, $mg\cos\theta$ into the slope
- Pull at θ above the horizontal on level ground: $R = mg - F\sin\theta$
- $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$

Notes

Resolving a force

- A force F at angle θ to a direction has component $F\cos\theta$ in that direction and $F\sin\theta$ perpendicular to it.

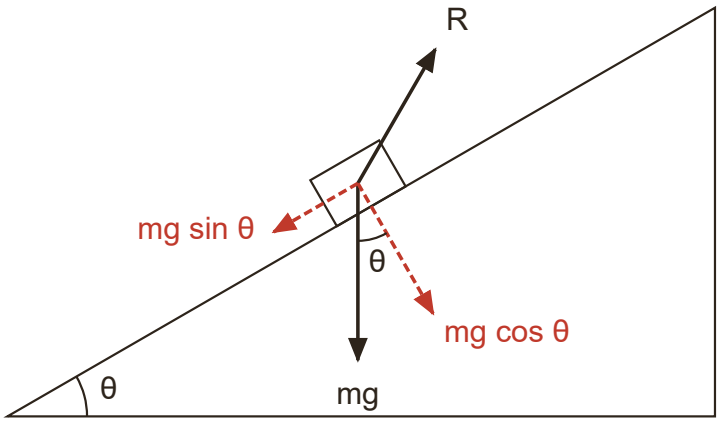


Component next to the angle: F cos θ; opposite the angle: F sin θ

- Resolve in two perpendicular directions: usually along the motion and perpendicular to it.
- A particle acted on by several coplanar forces is in equilibrium when the components add to zero in each of two perpendicular directions.
- A rope pulling at θ above the horizontal: $F\cos\theta$ drives the motion and $F\sin\theta$ partly lifts the object, so on level ground $R = mg - F\sin\theta$
- A push at θ below the horizontal presses the object into the ground: $R = mg + F\sin\theta$

Inclined planes

- Resolve parallel and perpendicular to the plane.
- On a slope at angle θ to the horizontal, the weight has components $mg\sin\theta$ down the line of greatest slope and $mg\cos\theta$ perpendicular to (into) the plane.



Resolve along and perpendicular to the slope: R = mg cos θ when nothing else pushes into the plane

- There is no acceleration perpendicular to the plane, so the forces there balance. With no other forces perpendicular to the plane, $R = mg\cos\theta$
- On a smooth slope with no other forces, $a = g\sin\theta$ down the slope, whatever the mass.
- A horizontal force P pushing a particle towards the slope has components $P\cos\theta$ up the slope and $P\sin\theta$ into the plane.
- 'Line of greatest slope' means the direction straight up or down the plane.

Exact values and checks

- If $\tan\alpha = \frac{3}{4}$, then $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$. If $\tan\alpha = \frac{5}{12}$, then $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$. Use these exact values.
- Check with $\theta = 0$: your equations should turn into the ones for level ground (no weight component along the plane, and $R = mg$).

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Particle on a smooth inclined plane

4 to 6 marks **Grade D**

1. Resolve perpendicular to the plane: $R = mg\cos\theta$
2. Resolve parallel to the plane and use $F = ma$
3. Use suvat along the slope for speeds and distances.

Example. A particle of mass 2 kg is released from rest on a smooth plane inclined at 30° to the horizontal.
(a) Find the magnitude of the normal reaction on the particle.
(b) Find the acceleration of the particle.
(c) Find the speed of the particle after it has moved 3.6 m down the plane.

Model answer.

(a) $R = 2g\cos 30°$ (M1) = 17 N (2 s.f.) or 17.0 N (3 s.f.) (A1)

(b) $2g\sin 30° = 2a$ (M1), $a = 4.9$ m s^{-2} (A1)

(c) $v^2 = 2(4.9)(3.6) = 35.28$ (M1), $v = 5.9$ m s^{-1} (2 s.f.) or 5.94 m s^{-1} (3 s.f.) (A1)

Pulled by a force at an angle on level ground

4 to 6 marks **Grade C**

1. Resolve horizontally: the component $F\cos\theta$ gives the acceleration.
2. Resolve vertically: $R + F\sin\theta = mg$
3. Give each answer to a sensible accuracy.

Example. A sledge of mass 20 kg is pulled across smooth horizontal ice by a rope inclined at 25° above the horizontal. The tension in the rope is 60 N. The sledge is modelled as a particle.
(a) Find the acceleration of the sledge.
(b) Find the magnitude of the normal reaction between the sledge and the ice.

Model answer.

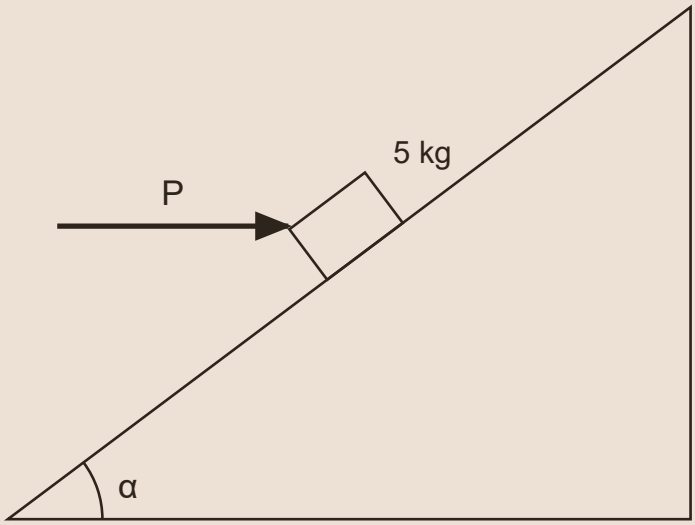
(a) $60\cos 25° = 20a$ (M1 A1), $a = 2.72$ m s^{-2} (3 s.f.) (A1)

(b) $R + 60\sin 25° = 20g$ (M1), $R = 170$ N (2 s.f.) or 171 N (3 s.f.) (A1)

Equilibrium on a smooth slope with a horizontal force 5 to 7 marks **Grade B**

1. Draw the weight, the normal reaction and the horizontal force.
2. Resolve parallel to the plane: $P\cos\alpha = mg\sin\alpha$
3. Resolve perpendicular to the plane: $R = mg\cos\alpha + P\sin\alpha$
4. Use exact trig values if $\tan\alpha$ is given.

Example. A particle of mass 5 kg is held in equilibrium on a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a horizontal force of magnitude P newtons. The line of action of the force lies in the vertical plane containing a line of greatest slope.



smooth plane, tan α = 3/4

(a) Find P.
(b) Find the magnitude of the normal reaction.

Model answer.

$\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

(a) $P\cos\alpha = 5g\sin\alpha$ (M1 A1), $P = 5g \times \frac{3}{4} = 36.75$, so 37 (2 s.f.) or 36.8 (3 s.f.) (A1)

(b) $R = 5g\cos\alpha + P\sin\alpha$ (M1 A1) $= 39.2 + 22.05 = 61.25$, so 61 N (2 s.f.) or 61.3 N (3 s.f.) (A1)

Multi-stage motion on a slope 7 to 10 marks **Grade A**

1. Stage 1: resolve parallel to the plane and use $F = ma$ to find the acceleration.
2. Use suvat to find the speed at the end of stage 1.
3. Stage 2: the force has gone, so find the new acceleration (on a smooth plane, $g\sin\theta$ down the slope).
4. Use the stage 1 final speed as the stage 2 initial speed.

Example. A box of mass 8 kg is pulled from rest up a line of greatest slope of a smooth plane inclined at 20° to the horizontal. The pulling force has magnitude 60 N and acts at 15° above the line of greatest slope, in the vertical plane containing that line. The box is modelled as a particle.
(a) Find the acceleration of the box.
(b) After 2 s the force is removed. Find the further distance the box moves up the plane before it comes to instantaneous rest.

Model answer.

(a) $60\cos 15° - 8g\sin 20° = 8a$ (M1 A1 A1), $a = 3.9$ m s^{-2} (2 s.f.) or 3.89 m s^{-2} (3 s.f.) (A1)

(b) Speed after 2 s: $v = 2a = 7.785...$ m s^{-1} (M1)

After the force is removed, deceleration $= g\sin 20°$
$= 3.351...$ m s^{-2} (M1)

$0 = 7.785^2 - 2(3.351)s$ (M1), $s = 9.0$ m (2 s.f.) or 9.04 m (3 s.f.) (A1)

Don’t lose marks

- Swapping sin and cos in the components of the weight.
- Assuming $R = mg$ on a slope, or $R = mg\cos\theta$ when another force has a component perpendicular to the plane.
- Resolving a horizontal force as if it acted along the slope.
- Confusing the slope angle with the angle between a force and the slope.
- Using rounded decimals for $\sin\alpha$ and $\cos\alpha$ when exact fractions are available.

More tips

Memory tricks

- Set $\theta = 0$ in your equations as a check: they should become the equations for level ground.
- Resolve perpendicular to a force you don't need, so that it drops out.
- A horizontal force holding a particle at rest on a smooth slope: $P = mg\tan\theta$
- The angle between the weight and the normal to the plane equals the slope angle θ, so the component into the plane is $mg\cos\theta$

Exam technique

- Draw a large diagram and mark the slope angle between the weight and the normal to the plane, as well as at the base.
- Label equations 'R(∥ plane)' and 'R(⊥ plane)', or write 'Resolving parallel to the plane'.
- Keep unrounded values between the parts of a multi-stage question.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Resolve a force into two components**
A force F at θ to a direction has components $F\cos\theta$ along it and $F\sin\theta$ perpendicular to it.

Grade D — **Resolve weight on an inclined plane**
On a slope at angle θ: $mg\sin\theta$ down the slope and $mg\cos\theta$ into the slope.

Grade C — **Find the acceleration on a smooth slope**
Use $F = ma$ parallel to the slope; the forces perpendicular to the slope balance.

Grade C — **Handle a pulling force at an angle**
The horizontal component drives the motion; the vertical component changes the normal reaction.

Grade B — **Solve equilibrium on a slope with a horizontal force**
Resolve the horizontal force and the weight parallel and perpendicular to the slope.

Grade A — **Solve multi-stage problems on inclined planes**
Find the new acceleration when a force is removed and link the stages with suvat.

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