Newton's third law, connected particles and pulleys

M8.4 · Mechanics · revision notes

Newton's third law says that when two bodies interact they exert equal and opposite forces on each other. You use it to model connected particles: a car towing a trailer, a person in a lift, and particles joined by a string over a pulley. These are common multi-part questions on Paper 3, often with a second stage after a string breaks or a particle hits the ground.

Key facts

- Newton's third law: equal and opposite forces on different bodies
- Each particle: $F = ma$ with the same a and the same T
- Inextensible string: same magnitude of acceleration
- Force on a pulley from two vertical strings: $2T$
- Whole system: external forces = total mass × a
- Smooth pulley and light string: same tension on both sides
- Person in a lift: $R - mg = ma$ (a upwards)
- String slack: $T = 0$

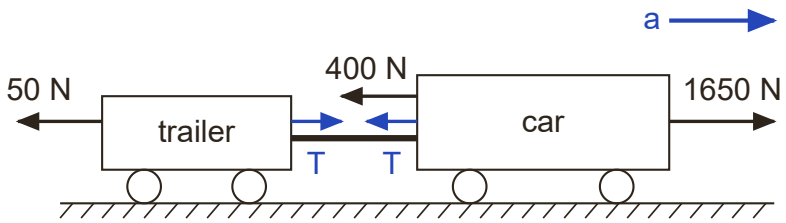
Notes

Newton's third law

- If body A exerts a force on body B, then B exerts a force on A that is equal in magnitude and opposite in direction.
- The two forces act on **different** bodies, so they never both appear on the force diagram for one body.
- Examples: a person pushes down on a lift floor and the floor pushes up on the person; a tow bar pulls the trailer forwards and pulls the car backwards with a force of the same size.

Connected particles moving together

- Objects joined by a rigid tow bar, or by a taut inextensible string, move with the same speed and the same magnitude of acceleration.
- **Whole system**: treat the objects as one particle of total mass, using only the external forces. The tension is internal, so it cancels. This gives a quickly.
- **Separate parts**: apply $F = ma$ to one object to find the tension (or reaction) between them.

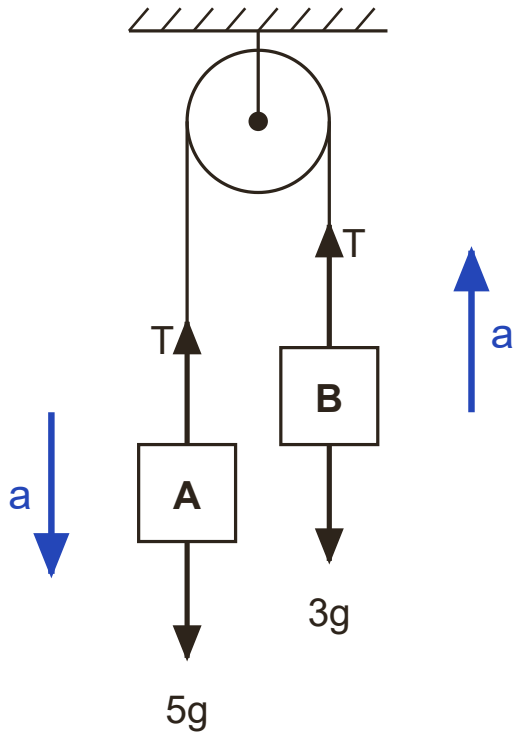


Tension T acts forwards on the trailer and backwards on the car: it cancels for the whole system

- A tow bar can be in **tension** (pulling) or in **thrust** (compression, pushing), for example when the car brakes and the trailer pushes on it. If you assume a tension and get a negative answer, the force is a thrust.
- Lift: for a person of mass m standing on the floor of a lift with upward acceleration a, $R - mg = ma$. For a lift of mass M and the person together, $T - (M + m)g = (M + m)a$

Pulleys

- For a light inextensible string over a smooth pulley, the tension is the same on both sides and both particles have the same magnitude of acceleration.



Same tension on both sides; write F = ma for each particle in its own direction

- Write $F = ma$ for each particle in its own direction of motion, then add the equations to eliminate T
- The force of the string on the pulley is the resultant of the two tensions: $2T$ for two vertical strings, or $T\sqrt{2}$ at 45° when one part is horizontal and the other vertical.
- When one particle hits the ground the string goes slack ($T = 0$). The other particle keeps the speed it had at that moment and then moves freely under gravity (or under friction, on a rough table).

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Car and trailer: acceleration and force in the tow bar

6 to 10 marks | Grade D

1. Whole system: driving force − total resistance = total mass × a
2. One part (usually the trailer): T − its resistance = its mass × a
3. If the driving force is removed, find the new (negative) acceleration from the whole system first.
4. A negative tension means the tow bar is in thrust.

Example. A car of mass 900 kg tows a trailer of mass 300 kg along a straight horizontal road by means of a light rigid tow bar. The resistances to motion are 400 N on the car and 50 N on the trailer. The engine of the car produces a driving force of 1650 N.
(a) Find the acceleration of the car and trailer.
(b) Find the tension in the tow bar.
The car's engine is now switched off. The resistances are unchanged.
(c) Find the magnitude of the force in the tow bar, and state whether it is a tension or a thrust.

Model answer.

(a) $1650 - 400 - 50 = 1200a$ (M1 A1), $a = 1$ m s^{-2} (A1)

(b) Trailer: $T - 50 = 300 \times 1$ (M1), $T = 350$ N (A1)

(c) Whole system: $-450 = 1200a$, so $a = -0.375$ m s^{-2} (M1 A1)

Trailer: $T - 50 = 300 \times (-0.375)$ (M1), $T = -62.5$, so the force has magnitude 62.5 N (A1) and is a thrust (A1)

Lift problems and Newton's third law

4 to 6 marks | Grade C

1. For the lift and passenger together: $T - (M + m)g = (M + m)a$
2. For the passenger alone: $R - mg = ma$
3. Newton's third law: the passenger pushes down on the floor with a force equal in size to R

Example. A lift of mass 500 kg carries a passenger of mass 70 kg. The lift is raised by a vertical cable and moves upwards with constant acceleration 1.2 m s^{-2}.
(a) Find the tension in the cable.
(b) Find the magnitude of the force exerted by the passenger on the floor of the lift.

Model answer.

(a) $T - 570g = 570 \times 1.2$ (M1 A1), $T = 570 \times 11 = 6270$ N (A1)

(b) Passenger: $R - 70g = 70 \times 1.2$ (M1), $R = 770$ N (A1); by Newton's third law the passenger exerts a force of 770 N on the floor (B1)

Particles hanging over a smooth pulley

6 to 10 marks **Grade B**

1. Write $F = ma$ for each particle: the heavier one accelerates down, the lighter one up.
2. Add the equations to eliminate T and find a; substitute back to find T
3. Force on the pulley: $2T$ for vertical strings.
4. After the heavier particle hits the ground, the lighter one moves freely under gravity with the speed it had at that moment.

Example. Particles A and B, of masses 5 kg and 3 kg, are attached to the ends of a light inextensible string that passes over a small smooth fixed pulley. The particles hang with the string taut and the hanging parts vertical. They are released from rest with A 1.5 m above the ground.
(a) Find the acceleration of the particles and the tension in the string.
(b) Find the magnitude of the force exerted on the pulley by the string.
(c) A hits the ground and does not rebound, and B does not reach the pulley. Find the further distance that B rises after A hits the ground.

Model answer.

(a) A: $5g - T = 5a$; B: $T - 3g = 3a$ (M1 A1 A1); adding, $2g = 8a$, so $a = 2.45$ m s^{-2} (A1); $T = 3(9.8 + 2.45) = 36.75$, so 37 N (2 s.f.) or 36.8 N (3 s.f.) (A1)

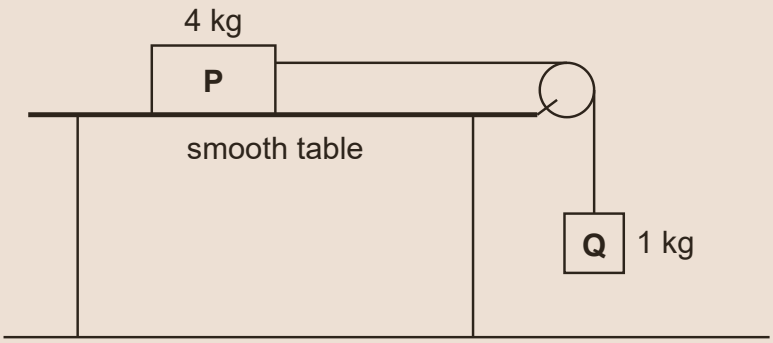
(b) $2T = 73.5$ N (B1)

(c) Speed when A lands: $v^2 = 2(2.45)(1.5) = 7.35$ (M1); then B moves freely under gravity: $0 = 7.35 - 2(9.8)s$ (M1), $s = 0.375$, so 0.38 m (2 s.f.) or 0.375 m (3 s.f.) (A1)

Particle on a table joined to a hanging particle 5 to 8 marks **Grade B**

1. Particle on the smooth table: $T = m_1 a$ horizontally.
2. Hanging particle: $m_2 g - T = m_2 a$
3. Solve for a and T
4. The force on the pulley is the resultant of a horizontal tension and a vertical tension: magnitude $T\sqrt{2}$, at 45° below the horizontal.

Example. A block P of mass 4 kg lies on a smooth horizontal table. It is attached to one end of a light inextensible string, which passes over a small smooth pulley fixed at the edge of the table. A particle Q of mass 1 kg hangs freely from the other end of the string. The system is released from rest with the string taut.
(a) Find the acceleration of P and the tension in the string.
(b) Find the magnitude of the force exerted on the pulley by the string.



Model answer.

(a) P: $T = 4a$; Q: $g - T = 1 \times a$ (M1 A1 A1); adding, $g = 5a$, so $a = 1.96$ m s^{-2} (A1)
and $T = 7.84$ N (A1)

(b) $\sqrt{T^2 + T^2} = T\sqrt{2}$ (M1) = 11 N (2 s.f.) or 11.1 N (3 s.f.) (A1)

Don’t lose marks

- Including the tension in the whole-system equation.
- Using different accelerations for particles joined by a taut inextensible string.
- Using the total mass with only one particle's forces.
- Assuming the tension equals the weight of a hanging particle that is accelerating.
- Forgetting that after the string goes slack the other particle keeps moving: it does not stop at once.
- In lift questions, leaving out the weight of the lift or of the passenger.

More tips

Memory tricks

- Two hanging masses $m_1 > m_2$: $a = \dfrac{(m_1 - m_2)g}{m_1 + m_2}$. Use it to check, but write the two equations in the exam.
- The tension always lies between the two weights: $m_2 g < T < m_1 g$
- Check a tow bar force using the other vehicle's equation: both should give the same value.
- Car and trailer slowing down with no driving force: if the trailer's resistance per kilogram is less than the car's, the tow bar is in thrust.

Exam technique

- Draw a separate force diagram for each particle and mark the direction of its acceleration.
- Label each equation, e.g. 'For A (↓): $5g - T = 5a$'.
- For 'how have you used the fact that the string is inextensible / the pulley is smooth?', answer: the particles have the same acceleration / the tension is the same on both sides of the pulley.
- Say 'tension' or 'thrust' when asked for the force in a tow bar or rod.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **State Newton's third law**
The forces between two bodies are equal in size, opposite in direction and act on different bodies.

Grade D — **Solve car and trailer problems**
Use the whole system to find the acceleration, then one part to find the tension or thrust.

Grade C — **Find the reaction force in a lift**
Apply $F = ma$ to the person; the force on the floor is equal and opposite (Newton's third law).

Grade C — **Solve problems with particles over a pulley**
Apply $F = ma$ to each particle, with the same acceleration and the same tension.

Grade B — **Find the force exerted on a pulley**
It is the resultant of the two tensions that act on the pulley.

Grade A — **Analyse motion after a string goes slack**
Once a particle hits the ground the tension is zero, and the other particle continues with the speed it had.

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