Newton's second law and vectors

M8.2 · Mechanics · revision notes

Newton's second law, $F = ma$: the resultant force on a particle equals its mass times its acceleration. You use it in one dimension (forces along and perpendicular to the motion) and in vector form, with forces given in $\mathbf{i}$, $\mathbf{j}$ form, usually together with the constant acceleration formulae.

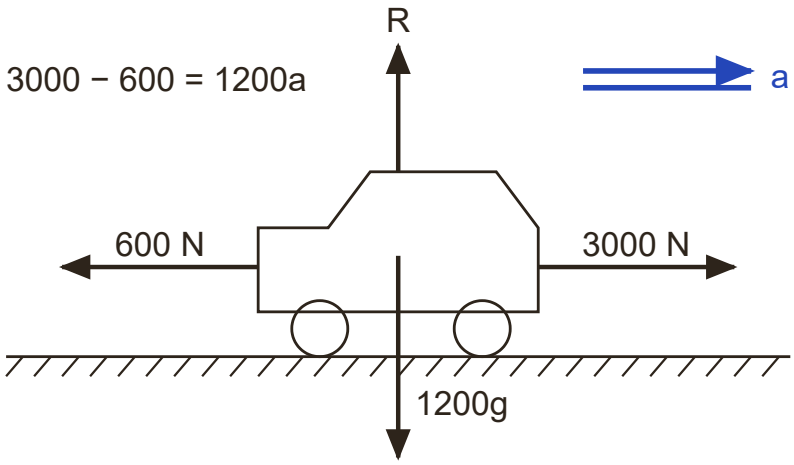
Key facts

- $F = ma$: resultant force = mass × acceleration
- Constant velocity ⇒ resultant force = 0
- Constant force ⇒ constant acceleration ⇒ suvat applies
- 1 N gives a mass of 1 kg an acceleration of 1 m s^{-2}
- Vector form: $\mathbf{F} = m\mathbf{a}$, $\mathbf{a} = \frac{1}{m}\mathbf{F}$
- The acceleration is in the direction of the resultant force

Notes

Newton's second law

- **Newton's second law**: resultant force = mass × acceleration, $F = ma$, with F in N, m in kg and a in m s^{-2}.
- The acceleration is in the direction of the resultant force.
- Write the equation in the direction of the acceleration: (forces in that direction) − (forces against it) $= ma$



Resultant force in the direction of the acceleration = ma

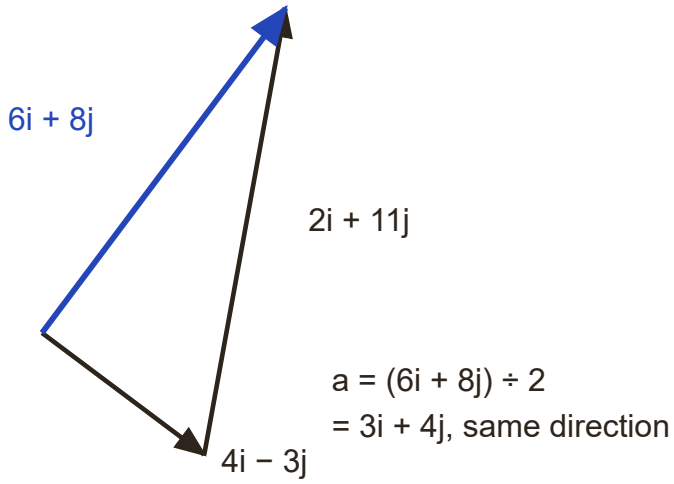
- Perpendicular to the motion there is no acceleration, so the forces balance; e.g. on level ground with no other vertical forces, $R = mg$
- If $a = 0$ the forces balance (Newton's first law): a car moving at constant speed has driving force = resistance.

Forces and motion together

- A constant resultant force gives a constant acceleration, so suvat can be used.
- Either forces → $\boldsymbol{a}$ → suvat (to find speeds, times and distances), or suvat → $\boldsymbol{a}$ → forces (to find an unknown force).
- When a force changes (an engine is switched off, a rope breaks), start a new stage with a new acceleration. The final velocity of one stage is the initial velocity of the next.

Vector form

- With forces as vectors: $\mathbf{F} = m\mathbf{a}$, where $\mathbf{F}$ is the vector sum of the forces.
- $\mathbf{a} = \frac{1}{m}\mathbf{F}$: divide each component by m. The acceleration is parallel to the resultant force.



The acceleration is parallel to the resultant force

- The magnitude of the acceleration is $|\mathbf{a}|$
- With constant forces, use vector suvat: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
- To find an unknown force: find $\mathbf{a}$ (e.g. $\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t}$), work out $m\mathbf{a}$, then subtract the forces you know.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find the acceleration from the forces

3 to 5 marks | Grade E

1. Draw a force diagram and mark the direction of motion.
2. Resultant force = forward forces − backward forces.
3. Use $F = ma$, then suvat if speeds or times are asked for.

Example. A car of mass 1200 kg moves along a straight horizontal road. The engine produces a constant driving force of 3000 N and the total resistance to motion is a constant 600 N.
(a) Find the acceleration of the car.
(b) The car starts from rest. Find its speed after 5 s.

Model answer.

(a) $3000 - 600 = 1200a$ (M1 A1), $a = 2$ m s^{-2} (A1)

(b) $v = 0 + 2 \times 5$ (M1) = 10 m s^{-1} (A1)

Find a force from the motion

4 to 5 marks | Grade D

1. Use suvat with the given speeds, distances or times to find a
2. A negative a means a deceleration.
3. Use $F = ma$ with the resultant force to find the unknown.

Example. A particle of mass 0.5 kg moves in a straight line on a horizontal surface. Its speed decreases from 12 m s^{-1} to 4 m s^{-1} while it travels 16 m. The only horizontal force on the particle is a constant resistance of magnitude R newtons. Find R

Model answer.

$4^2 = 12^2 + 2a(16)$ (M1), $a = -4$ m s^{-2} (A1)

$-R = 0.5 \times (-4)$ (M1), so $R = 2$ (A1)

Acceleration from forces in vector form

5 to 7 marks | Grade C

1. Add the force vectors to get the resultant.
2. Divide by the mass to get $\mathbf{a}$
3. Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for the velocity, and magnitudes for speeds.

Example. A particle P of mass 2 kg is acted on by two constant forces, $(4\mathbf{i} - 3\mathbf{j})$ N and $(2\mathbf{i} + 11\mathbf{j})$ N, and no others.
(a) Find the magnitude of the acceleration of P.
(b) P starts from rest. Find the speed of P after 3 s.

Model answer.

(a) Resultant $= (6\mathbf{i} + 8\mathbf{j})$ N (M1); $\mathbf{a} = \frac{1}{2}(6\mathbf{i} + 8\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$ m s^{-2} (M1 A1); $|\mathbf{a}| = 5$ m s^{-2} (A1)

(b) $\mathbf{v} = 3(3\mathbf{i} + 4\mathbf{j})$
$= (9\mathbf{i} + 12\mathbf{j})$ m s^{-1} (M1), so the speed is 15 m s^{-1} (A1)

Find an unknown force from vector motion

4 to 6 marks | Grade B

1. Find the acceleration vector from the change in velocity: $\mathbf{a} = \dfrac{\mathbf{v} - \mathbf{u}}{t}$
2. Resultant force $= m\mathbf{a}$
3. Unknown force = resultant − known forces; find its magnitude if asked.

Example. A particle of mass 4 kg moves under the action of two constant forces, $\mathbf{F}_1 = (2\mathbf{i} + 5\mathbf{j})$ N and $\mathbf{F}_2$. The particle starts from rest and 2 seconds later its velocity is $(6\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find the magnitude of $\mathbf{F}_2$

Model answer.

$$\mathbf{a} = \frac{6\mathbf{i} - 4\mathbf{j}}{2}$$
$$= (3\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2} \quad \text{(M1 A1)}$$

Resultant $= 4(3\mathbf{i} - 2\mathbf{j})$
$= (12\mathbf{i} - 8\mathbf{j})$ N (M1)

$$\mathbf{F}_2 = (12\mathbf{i} - 8\mathbf{j}) - (2\mathbf{i} + 5\mathbf{j})$$
$$= (10\mathbf{i} - 13\mathbf{j}) \text{ N} \quad \text{(M1 A1)}$$

$$|\mathbf{F}_2| = \sqrt{269}$$
$$= 16.4 \text{ N (3 s.f.)} \quad \text{(A1)}$$

Don’t lose marks

- Using one force instead of the resultant force in $F = ma$
- Forgetting the resistance or the friction.
- Writing $F = mg$ instead of $F = ma$
- Adding the magnitudes of forces instead of adding the forces as vectors.
- Using a mass in grams, or giving a weight in kg.

More tips

Memory tricks

- Write Newton's second law with a direction arrow, e.g. 'N2L (→): 3000 − 600 = 1200a', so the signs are clear.
- A particle that starts from rest under a constant force moves in a straight line in the direction of that force.
- If a comes out negative, the particle is decelerating in your positive direction: check that this makes sense.

Exam technique

- State the law and the direction each time: method marks are given for a correct N2L equation with the right number of terms.
- Give an acceleration as a vector unless the question asks for its magnitude.
- Keep full calculator values between parts and round only the final answer.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Apply $F = ma$ in a straight line**
The resultant force in the direction of motion equals mass × acceleration.

Grade D — **Combine $F = ma$ with suvat**
Find a from the forces and then use suvat, or find a from suvat and then find a force.

Grade C — **Use $F = ma$ with forces as vectors**
Add the force vectors and divide by the mass to get the acceleration vector.

Grade B — **Find an unknown force from vector motion**
Find $\mathbf{a}$ from the motion, work out the resultant force $m\mathbf{a}$, then subtract the known forces.

Grade A — **Solve problems where the forces change**
When a force is removed or changes, find the new acceleration and link the stages by the velocity.

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