Forces, resultants and equilibrium of a particle

M8.1, M8.4, M8.5 · Mechanics · revision notes

What forces are, the common types of force, Newton's first law, finding the resultant of several forces (as vectors or by resolving), and using the fact that a particle in equilibrium has zero resultant force. This is the foundation for every forces question on Paper 3.

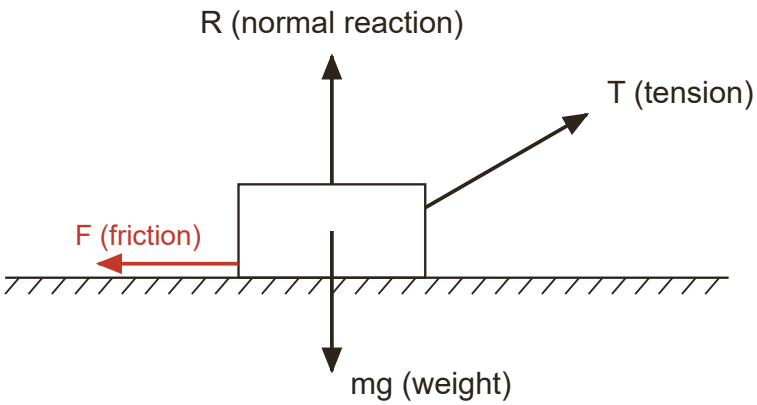
Key facts

- Newton's first law: zero resultant force ⇔ at rest or constant velocity
- Resultant = vector sum of all the forces
- Force F at θ to the horizontal: components $F\cos\theta$ (horizontal) and $F\sin\theta$ (vertical)
- Tension pulls along a string; thrust pushes along a rod
- Weight $W = mg$, vertically downwards
- $|x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$
- Equilibrium: resolve in two perpendicular directions, each sum = 0

Notes

Forces and Newton's first law

- A force is a push or a pull. It is a vector, measured in newtons (N).
- **Newton's first law**: a particle stays at rest, or moves with constant velocity, unless a resultant force acts on it.
- So a particle in **equilibrium** (zero resultant force) is either at rest or moving with constant velocity.
- Forces you need: **weight** mg, vertically downwards; **normal reaction**, perpendicular to a surface; **tension**, pulling along a string or rod; **thrust** (compression), pushing along a rod; **friction**, along a rough surface; **resistance** to motion, such as air resistance; the **driving force** of an engine.



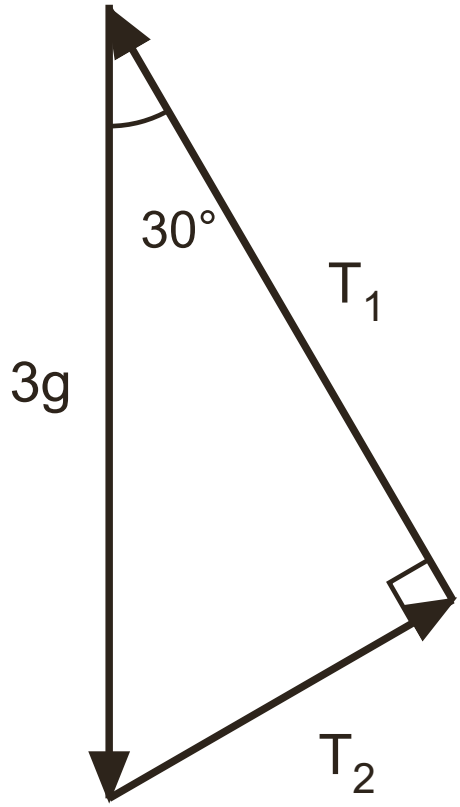
Draw every force on the object, each as an arrow in its own direction

Resultant forces

- The resultant is the single force that has the same effect as all the forces together. Add the forces as vectors.
- In $\mathbf{i}$, $\mathbf{j}$ form, add the components: $(3\mathbf{i} + 2\mathbf{j}) + (\mathbf{i} - 5\mathbf{j}) = 4\mathbf{i} - 3\mathbf{j}$
- The magnitude of $x\mathbf{i} + y\mathbf{j}$ is $\sqrt{x^2 + y^2}$; its angle with $\mathbf{i}$ comes from $\tan\theta = \frac{y}{x}$. Sketch it to get the right quadrant.
- For forces given by magnitude and direction, resolve each one into two perpendicular components, add the components, then recombine.
- A resultant 'parallel to $a\mathbf{i} + b\mathbf{j}$' has its components in the ratio $a : b$

Equilibrium of a particle

- In equilibrium the resultant force is zero: resolve in two perpendicular directions and set each sum to zero.
- For forces given as vectors, the sum of the forces is $0\mathbf{i} + 0\mathbf{j}$
- Choose directions that make the working easy, e.g. horizontal and vertical, or perpendicular to a force you don't know.
- Three forces in equilibrium form a closed triangle when drawn end to end, so you can also use the sine rule or the cosine rule.



Three forces in equilibrium, drawn nose to tail, close into a triangle:
here $T_1 = 3g \cos 30°$, $T_2 = 3g \cos 60°$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, dM = depends on the earlier method mark.

Resultant of forces in vector form

4 to 7 marks **Grade D**

1. Add the $\mathbf{i}$ components and the $\mathbf{j}$ components.
2. Find the magnitude using Pythagoras.
3. Find the angle using $\tan$, and say which direction it is measured from.
4. For equilibrium, the extra force needed is minus the resultant.

Example. Three forces, $(3\mathbf{i} - 2\mathbf{j})$ N, $(-5\mathbf{i} + 6\mathbf{j})$ N and $(4\mathbf{i} + 3\mathbf{j})$ N, act on a particle.
(a) Find the magnitude of the resultant force.
(b) Find the angle between the resultant force and the vector $\mathbf{i}$.
(c) A fourth force $\mathbf{F}$ is added so that the particle is in equilibrium. Find $\mathbf{F}$

Model answer.

(a) Resultant $= (2\mathbf{i} + 7\mathbf{j})$ N (M1 A1); magnitude $= \sqrt{2^2 + 7^2} = \sqrt{53} = 7.28$ N (3 s.f.) (M1 A1)

(b) $\tan\theta = \dfrac{7}{2}$ (M1), $\theta = 74.1°$ (A1)

(c) $\mathbf{F} = (-2\mathbf{i} - 7\mathbf{j})$ N (B1)

Resultant parallel to a given vector

3 to 5 marks **Grade C**

1. Write the resultant in terms of the unknown.
2. Parallel to $a\mathbf{i} + b\mathbf{j}$ means ($\mathbf{j}$ component) $= \dfrac{b}{a}$ × ($\mathbf{i}$ component).
3. Solve, then find anything else asked for, such as the magnitude.

Example. Two forces, $(-\mathbf{i} + 8\mathbf{j})$ N and $(p\mathbf{i} + p\mathbf{j})$ N, where p is a constant, act on a particle. The resultant of the two forces is parallel to $(\mathbf{i} + 2\mathbf{j})$.
(a) Find the value of p.
(b) Find the magnitude of the resultant.

Model answer.

(a) Resultant $= (p-1)\mathbf{i} + (p+8)\mathbf{j}$ (M1); $p + 8 = 2(p-1)$ (M1), so $p = 10$ (A1)

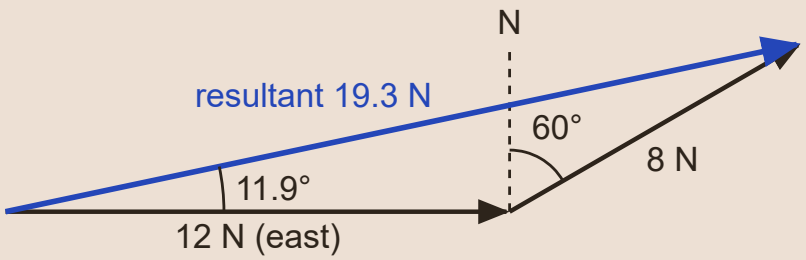
(b) Resultant $= 9\mathbf{i} + 18\mathbf{j}$, magnitude $= \sqrt{405} = 20.1$ N (3 s.f.) (M1 A1)

Resultant of forces given by magnitude and direction 5 to 7 marks **Grade B**

1. Choose two perpendicular directions (e.g. east and north).
2. Resolve every force in both directions and add.
3. Recombine: magnitude by Pythagoras, direction by $\tan$; give a bearing if the question uses bearings.

Example. A particle is acted on by two horizontal forces: a force of 12 N acting due east and a force of 8 N acting on a bearing of 060°. Find the magnitude of the resultant force and the bearing on which it acts.

Model answer.



East: $12 + 8\sin 60° = 18.93$ N; north: $8\cos 60° = 4$ N (M1 A1)

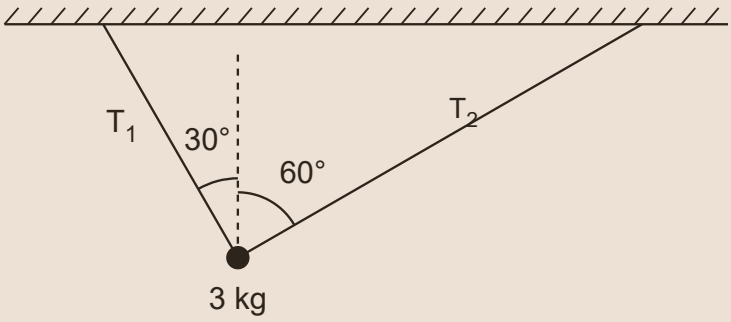
Magnitude $= \sqrt{18.93^2 + 4^2}$ (M1) = 19.3 N (3 s.f.) (A1)

Angle north of east $= \tan^{-1}\dfrac{4}{18.93} = 11.9°$ (M1), so the bearing is 078° to the nearest degree (A1)

Particle held in equilibrium by two strings 5 to 7 marks **Grade A**

1. Draw the force diagram: the weight and the two tensions.
2. Resolve horizontally: the horizontal components of the tensions balance.
3. Resolve vertically: the vertical components of the tensions balance the weight.
4. Solve the simultaneous equations.

Example. A particle of mass 3 kg hangs in equilibrium, supported by two light inextensible strings on opposite sides of the vertical through the particle. One string makes an angle of 30° with the vertical and has tension T_1 N; the other makes an angle of 60° with the vertical and has tension T_2 N. Find T_1 and T_2.



Model answer.

Horizontally: $T_1 \sin 30° = T_2 \sin 60°$ (M1 A1), so $T_1 = \sqrt{3}\,T_2$

Vertically: $T_1 \cos 30° + T_2 \cos 60° = 3g$ (M1 A1)

$\frac{3}{2}T_2 + \frac{1}{2}T_2 = 3g$, so $T_2 = 1.5g = 14.7$ (dM1 A1) and $T_1 = 14.7\sqrt{3}$ = 25 (2 s.f.) or 25.5 (3 s.f.) (A1)

Don’t lose marks

- Leaving a force off the diagram, especially the weight or the normal reaction.
- Swapping sin and cos when resolving.
- Giving an angle without saying what it is measured from.
- Assuming the normal reaction always equals mg
- Thinking equilibrium means 'at rest' only: moving with constant velocity is equilibrium too.

More tips

Memory tricks

- The component 'next to' the angle uses cos; the component 'opposite' the angle uses sin.
- Resolve perpendicular to a force you don't know, to remove it from the equation.
- Sense checks: a tension cannot be negative, and a resultant can never be larger than the sum of the magnitudes.
- Two strings at equal angles either side of the vertical have equal tensions.

Exam technique

- Draw and label a force diagram before resolving, even when one is given.
- Show the direction you resolve in, e.g. 'R(→)' or 'Resolving vertically'.
- Work with exact values such as $3g$ and $\sqrt{3}$, rounding only at the end; give answers using $g = 9.8$ to 2 or 3 s.f.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Draw a clear, labelled force diagram**
Show the weight, normal reaction, tension or thrust, friction and any applied forces, each with an arrow.

Grade E — **Add forces given in vector form**
Add the $\mathbf{i}$ components and the $\mathbf{j}$ components separately to get the resultant.

Grade D — **Find the magnitude and direction of a force**
Use Pythagoras for the magnitude and $\tan\theta$ for the angle with $\mathbf{i}$

Grade C — **Use equilibrium to find unknown forces**
In equilibrium the resultant force is zero, so the components in each direction add to zero.

Grade B — **Resolve forces given by magnitude and direction**
A force F at angle θ to a direction has components $F\cos\theta$ along it and $F\sin\theta$ perpendicular to it.

Grade A — **Solve equilibrium problems with strings at angles**
Resolve horizontally and vertically and solve the simultaneous equations for the tensions.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.