Constant acceleration (suvat)

M7.3 · Mechanics · revision notes

The five constant acceleration formulae (suvat) for motion in a straight line, how to derive them, and their vector form in two dimensions. They are used in almost every mechanics question on Paper 3, often together with forces.

Key facts

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$
- $s = \frac{1}{2}(u + v)t$
- $v^2 = u^2 + 2as$
- Vector form: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
- Only for constant acceleration

Notes

The five formulae

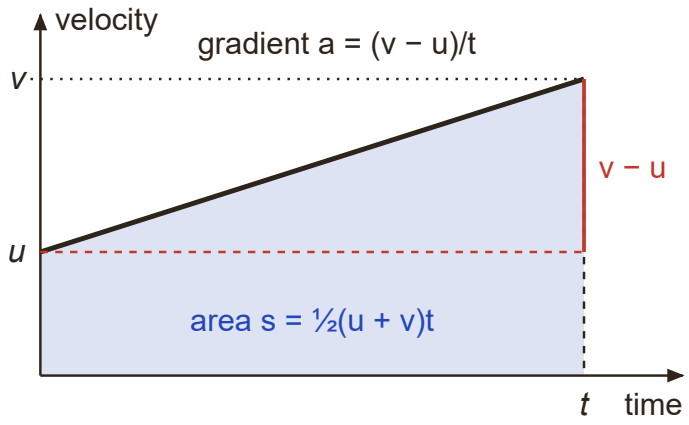
- They apply only when the acceleration is **constant**: in a straight line, or in vector form when the acceleration vector is constant.
- s = displacement, u = initial velocity, v = final velocity, a = acceleration, t = time.
- $v = u + at$ (no s); $s = ut + \frac{1}{2}at^2$ (no v); $s = vt - \frac{1}{2}at^2$ (no u); $s = \frac{1}{2}(u + v)t$ (no a); $v^2 = u^2 + 2as$ (no t).
- Method: write down s, u, v, a, t, fill in the three you know, mark the one you want, and use the formula that leaves out the fifth.
- Key phrases: 'from rest' → $u = 0$; 'comes to rest' → $v = 0$; 'uniform' or 'constant' acceleration → suvat applies.

Signs and directions

- Choose a positive direction. s, u, v and a are vectors and each takes a sign; t is positive.
- s is the displacement from the starting point, not the distance travelled. If the object turns round, find when $v = 0$ and treat each part separately.
- A quadratic in t can have two positive roots: the object is at that displacement twice. Reject negative roots.

Deriving the formulae

- Draw a velocity–time graph: a straight line from u to v over time t. The gradient gives $a = \dfrac{v-u}{t}$, so $v = u + at$
- The area under it is a trapezium, so $s = \dfrac{1}{2}(u+v)t$



Constant acceleration: the gradient gives v = u + at, the area gives s = ½(u + v)t

- Substitute $v = u + at$ into $s = \dfrac{1}{2}(u+v)t$ to get $s = ut + \dfrac{1}{2}at^2$; substitute $u = v - at$ to get $s = vt - \dfrac{1}{2}at^2$
- Substitute $t = \dfrac{v-u}{a}$ into $s = \dfrac{1}{2}(u+v)t$ to get $2as = v^2 - u^2$, so $v^2 = u^2 + 2as$

Vector form

- With constant acceleration $\mathbf{a}$: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \dfrac{1}{2}\mathbf{a}t^2$, where $\mathbf{r}_0$ is the starting position.
- Collect the $\mathbf{i}$ and $\mathbf{j}$ terms; the speed is $|\mathbf{v}|$. 'Moving parallel to $\mathbf{i}$' means the $\mathbf{j}$ component of $\mathbf{v}$ is zero.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, A1* = the solution must be fully correct, with every step shown.

Find a quantity using suvat

2 to 4 marks | **Grade E**

1. Write down s, u, v, a, t with the values you know, taking the direction of motion as positive.
2. Choose the formula without the quantity you neither know nor want.
3. Substitute, solve and give units.

Example. A particle moves in a straight line with constant acceleration. It passes the point A with speed 4 m s^{-1} and, 6 s later, passes the point B with speed 16 m s^{-1}.
(a) Find the acceleration of the particle.
(b) Find the distance AB

Model answer.

(a) $16 = 4 + 6a$ (M1), $a = 2$ m s^{-2} (A1)

(b) $s = \frac{1}{2}(4 + 16) \times 6$ (M1) = 60 m (A1)

Two objects: when and where do they meet?

5 to 7 marks | **Grade C**

1. Use the same origin and the same time t for both objects.
2. Write an expression for each displacement in terms of t
3. Equate the displacements and solve; $t = 0$ is just the start.
4. Substitute back to find the distance.

Example. Car A passes a point O with speed 8 m s^{-1} and moves with constant acceleration 2 m s^{-2}. At the same instant car B passes O with constant speed 20 m s^{-1}, moving in the same direction along the same straight road. Find the time taken for A to catch up with B, and the distance from O at which this happens.

Model answer.

$s_A = 8t + \frac{1}{2}(2)t^2$

$= 8t + t^2$ (M1 A1); $s_B = 20t$ (B1)

$8t + t^2 = 20t$, so $t(t - 12) = 0$ (M1), and $t = 12$ s (A1)

Distance $= 20 \times 12 = 240$ m (A1)

Derive a constant acceleration formula

2 to 4 marks | **Grade B**

1. Start from what you are told you may use (a graph, or two given formulae).
2. Rearrange one formula to make the unwanted variable the subject.
3. Substitute into the other and simplify to the required form, showing every line.

Example. Starting from the formulae $v = u + at$ and $s = \frac{1}{2}(u + v)t$, show that $v^2 = u^2 + 2as$

Model answer.

$t = \dfrac{v - u}{a}$ (M1)

$s = \dfrac{1}{2}(u + v) \times \dfrac{v - u}{a}$ (M1)

$2as = (v + u)(v - u)$

$= v^2 - u^2$ so $v^2 = u^2 + 2as$ (A1*)

Constant acceleration in vector form

5 to 8 marks | **Grade A**

1. Write $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and collect the $\mathbf{i}$ and $\mathbf{j}$ components.
2. Turn conditions into equations: 'parallel to $\mathbf{i}$' means the $\mathbf{j}$ component is zero.
3. Use $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ for the displacement, adding the starting position if it is not O
4. Find distances and speeds as magnitudes.

Example. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}. At time $t = 0$, P is at the origin O and has velocity $(-4\mathbf{i} + 5\mathbf{j})$ m s^{-1}.
(a) Find the velocity of P when $t = 3$.
(b) Find the value of t when P is moving parallel to $\mathbf{i}$.
(c) Find the distance OP when $t = 4$

Model answer.

(a) $\mathbf{v} = (-4\mathbf{i} + 5\mathbf{j}) + 3(2\mathbf{i} - 3\mathbf{j})$ (M1) $= (2\mathbf{i} - 4\mathbf{j})$ m s^{-1} (A1)

(b) $\mathbf{v} = (-4 + 2t)\mathbf{i} + (5 - 3t)\mathbf{j}$; $5 - 3t = 0$ (M1), so $t = \dfrac{5}{3}$ (A1)

(c) $\mathbf{r} = 4(-4\mathbf{i} + 5\mathbf{j}) + \frac{1}{2}(16)(2\mathbf{i} - 3\mathbf{j})$ (M1) $= -4\mathbf{j}$ (A1), so $OP = 4$ m (A1)

Don't lose marks

- Using suvat when the acceleration is not constant (for example $a = 3t$): use calculus instead.
- Forgetting that a deceleration makes a negative when the velocity is positive.

- Treating s as the distance travelled when the object changes direction.
- Not converting km h^{-1} to m s^{-1} first.
- Forgetting to take the square root after using $v^2 = u^2 + 2as$
- Rounding in an early stage and carrying the rounded value into later stages.

More tips

Memory tricks

- Missing-letter rule: find the one of s, u, v, a, t that is neither given nor wanted, and use the formula without it.
- 'Starts from rest' $\rightarrow u = 0$; 'comes to rest' $\rightarrow v = 0$; 'greatest height' $\rightarrow v = 0$
- Stopping distance from speed u with deceleration d: $\dfrac{u^2}{2d}$
- Sense check: times must be positive, and a car's stopping distance should be tens of metres, not kilometres.

Exam technique

- Write the suvat list at the start of every calculation: it earns method marks and prevents mix-ups.
- In a two-stage problem, label the stages and carry the exact final velocity of stage 1 into stage 2.
- In 'show that' derivations include every algebraic step: the final A1* is lost if a line is skipped.
- Give answers to 3 significant figures, or to 2 or 3 s.f. if you have used $g = 9.8$

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Choose and use the correct suvat formula**
List s, u, v, a, t, then pick the formula that links the three you know with the one you want.

Grade D — **Handle deceleration and direction with signs**
Choose a positive direction; a deceleration has the opposite sign to the velocity.

Grade C — **Solve two-stage and two-object problems**
The final velocity of one stage is the initial velocity of the next; objects meet when their displacements are equal.

Grade B — **Derive the constant acceleration formulae**
Use the gradient and area of a straight-line velocity–time graph, then eliminate variables.

Grade B — **Interpret both roots of a quadratic in t**
Two positive roots mean the object is at that displacement twice, for example on the way out and on the way back.

Grade A — **Use suvat in vector form**
Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{i}$ and $\mathbf{j}$ components.

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