Kinematics graphs

M7.2 · Mechanics · revision notes

Displacement–time and velocity–time graphs for motion in a straight line: what the gradient and the area mean and how to use them. Expect to sketch a velocity–time graph from a description, then use its gradients and areas to find an unknown time, speed or distance.

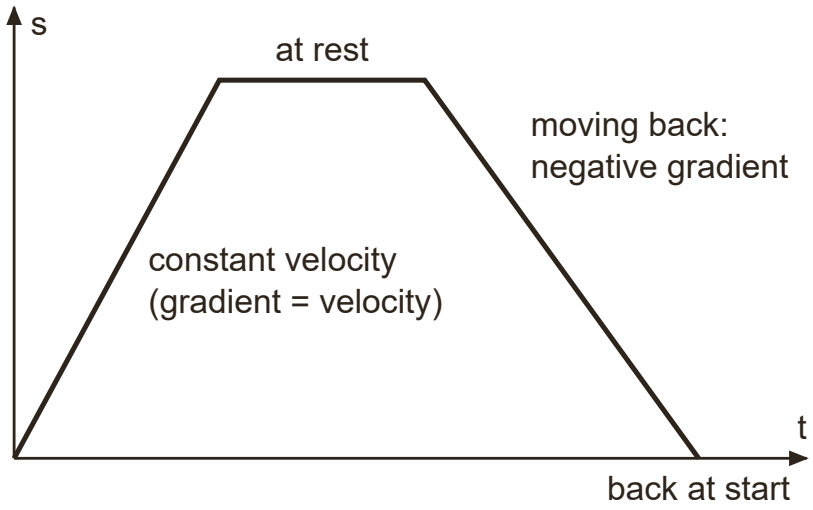
Key facts

- Displacement–time graph: gradient = velocity
- Velocity–time graph: area = displacement
- Trapezium area $= \frac{1}{2}(a+b)h$
- Velocity–time graph: gradient = acceleration
- Distance = total area, with every part counted as positive
- Lines crossing on a velocity–time graph: equal velocities, not the same position

Notes

Displacement–time graphs

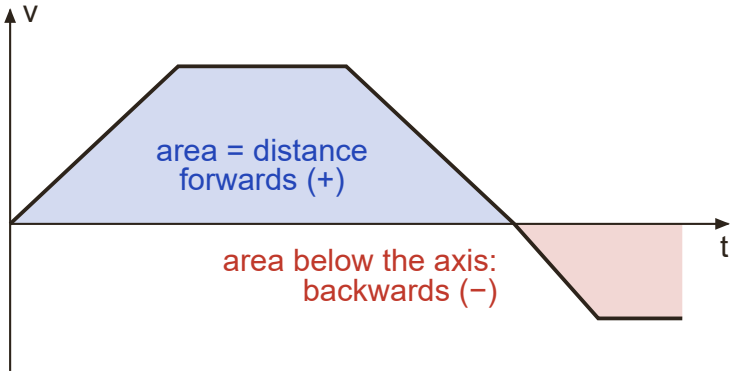
- The **gradient** is the velocity. A steeper line means a greater speed.
- A horizontal line means the object is at rest. A straight sloping line means constant velocity. A curve means the velocity is changing.
- A negative gradient means the object is moving in the negative direction, for example back towards the start.
- Where the graph meets the time axis, the object is back at its starting point (displacement zero).



Displacement–time: the gradient is the velocity

Velocity–time graphs

- The **gradient** is the acceleration; a negative gradient is a deceleration.
- The **area** between the graph and the time axis is the displacement.
- A horizontal line means constant velocity; a straight sloping line means constant acceleration.
- Below the time axis the velocity is negative: the object moves in the negative direction and that area counts as negative displacement.
- Distance travelled = total area, counting every part as positive. Displacement = area above the axis − area below it.



Velocity–time: gradient = acceleration, area = displacement (below the axis counts as negative)

- Areas you need: triangle $\frac{1}{2}bh$, rectangle bh, trapezium $\frac{1}{2}(a+b)h$

Solving problems with graphs

- Sketch the velocity–time graph first, even if the question does not ask for it. Label speeds and times, using letters such as T or V for unknowns.
- Form equations from gradients (accelerations) and areas (distances), then solve.
- The time to reach speed V from rest with acceleration a is $\frac{V}{a}$
- For two objects, draw both graphs on the same axes. They are level when their displacements from the same starting point are equal. Where two velocity–time lines cross, the objects have equal velocities, not equal positions.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

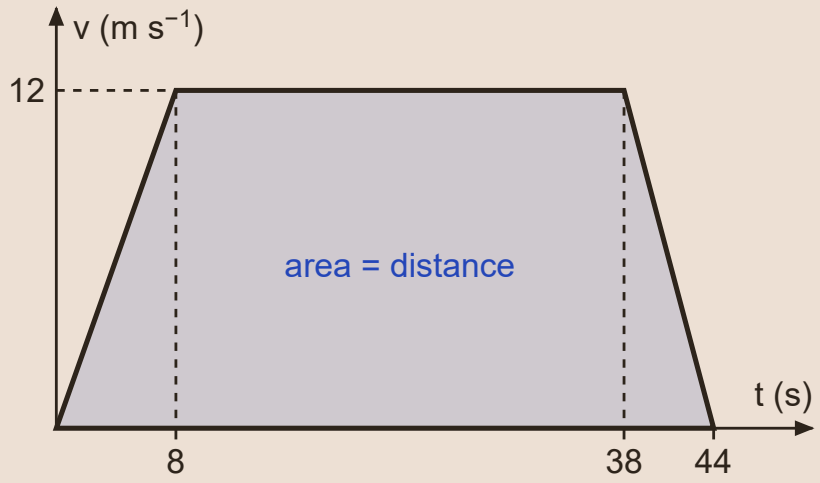
Sketch a velocity–time graph and find the distance

3 to 5 marks | **Grade E**

1. Split the journey into stages; each constant-acceleration stage is a straight line.
2. Label the axes (v in m s^{-1}, t in s) and mark every key speed and time.
3. Find the distance as the area, split into triangles and rectangles or treated as a trapezium.

Example. A car starts from rest and accelerates uniformly to 12 m s^{-1} in 8 s. It travels at 12 m s^{-1} for 30 s and then decelerates uniformly, coming to rest after a further 6 s.
(a) Sketch a velocity–time graph for the motion.
(b) Find the total distance travelled by the car.

Model answer.



(a) As in the diagram: a trapezium shape starting and finishing on the time axis (B1), with 12, 8, 38 and 44 marked (B1)

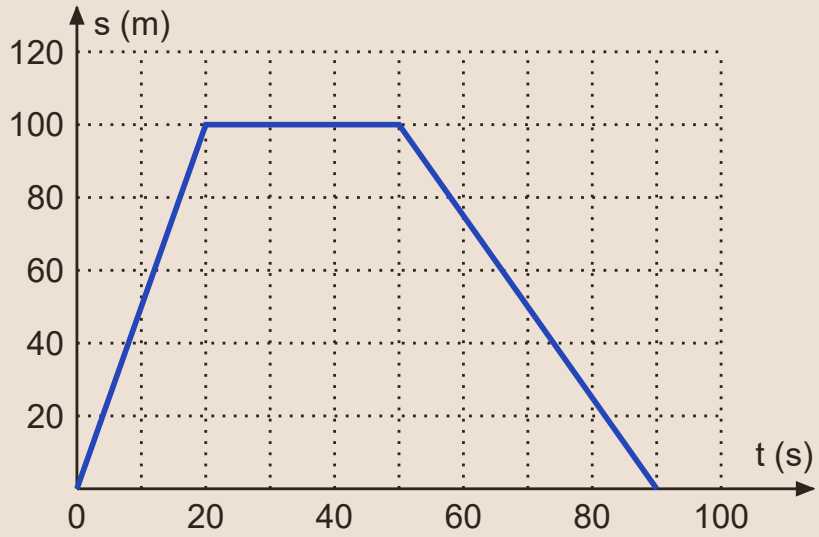
(b) Area $= \frac{1}{2}(30 + 44) \times 12$ (M1) = 444 m (A1)

Interpret a displacement–time graph

3 to 6 marks | **Grade D**

1. Find the gradient of each straight section: that is the velocity.
2. A horizontal section means at rest; a negative gradient means moving back.
3. Average speed = total distance ÷ total time; average velocity = final displacement ÷ total time.

Example. The displacement–time graph for a runner is made of three straight line segments joining the points (0, 0), (20, 100), (50, 100) and (90, 0), where t is the time in seconds and s is the displacement in metres from the start.



(a) Find the velocity of the runner in each of the three stages.
(b) Find the average speed of the runner over the 90 seconds.

Model answer.

(a) $\frac{100}{20}$ = 5 m s^{-1} (B1); 0 m s^{-1}, at rest (B1); $\frac{0-100}{90-50} = -2.5$ m s^{-1}, i.e. 2.5 m s^{-1} back towards the start (M1 A1)

(b) $\frac{100+100}{90}$ (M1) = 2.22 m s^{-1} (3 s.f.) (A1)

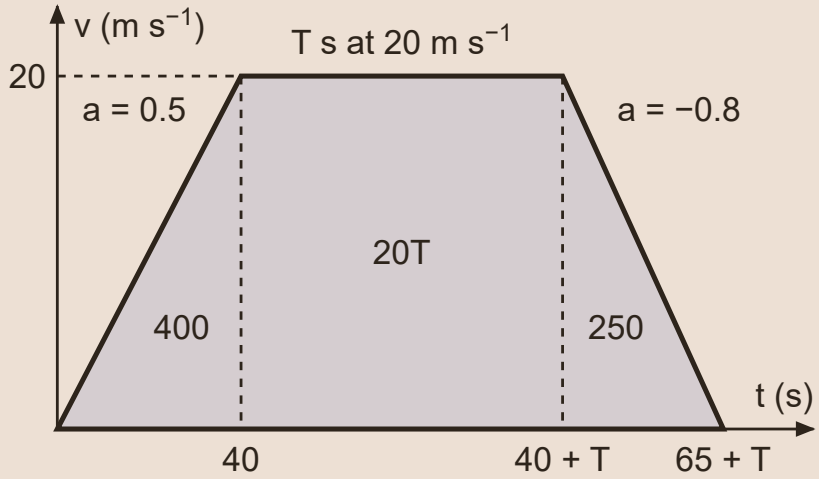
Find an unknown time from the total distance

4 to 6 marks | **Grade C**

1. Sketch the graph and find the time for each accelerating or decelerating stage, using gradient = acceleration.
2. Write the total area in terms of the unknown.
3. Set the area equal to the total distance, in metres, and solve.

Example. A train travels between two stations that are 3 km apart. It starts from rest and accelerates uniformly at 0.5 m s^{-2} until its speed is 20 m s^{-1}. It travels at 20 m s^{-1} for T seconds and then decelerates uniformly at 0.8 m s^{-2}, coming to rest at the second station. Find the value of T

Model answer.



Time accelerating $= \dfrac{20}{0.5} = 40$ s (B1); time decelerating $= \dfrac{20}{0.8} = 25$ s (B1)

$$\frac{1}{2} \times 40 \times 20 + 20T + \frac{1}{2} \times 25 \times 20 = 3000$$ (M1 A1)

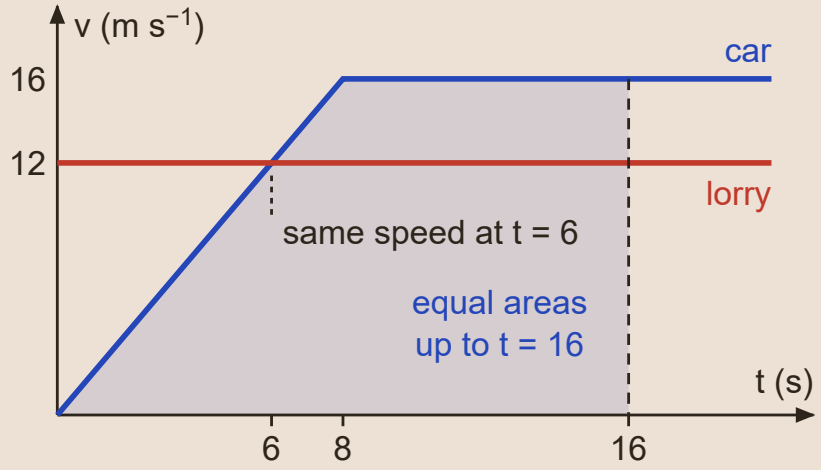
$400 + 20T + 250 = 3000$, so $T = 117.5$ (A1)

Two vehicles: when does one catch the other? 5 to 7 marks **Grade B**

1. Sketch both velocity–time graphs on the same axes.
2. Write each vehicle's displacement (area) at time t, checking which stage each vehicle is in.
3. Set the displacements equal and solve; check that the time is in the stage you assumed.

Example. A car is waiting at traffic lights. When the lights turn green, the car accelerates uniformly at 2 m s^{-2} until it reaches a speed of 16 m s^{-1}, and then continues at that speed. At the moment the car starts, a lorry passes it, travelling at a constant 12 m s^{-1} in the same direction. Find the time taken for the car to catch up with the lorry.

Model answer.



The car reaches 16 m s^{-1} after $\frac{16}{2} = 8$ s, having travelled $\frac{1}{2} \times 8 \times 16 = 64$ m; the lorry has then travelled 96 m, so the car is still behind (B1)

For $t > 8$, car's displacement $= 64 + 16(t - 8)$ (M1); lorry's displacement $= 12t$ (B1)

$64 + 16(t - 8) = 12t$ (M1), so $4t = 64$ and $t = 16$ s (A1)

Don’t lose marks

- Thinking that where two velocity–time lines cross, one object overtakes the other.
- Using the area under a displacement–time graph: it has no meaning here.
- Subtracting the area below the axis when the question asks for the distance travelled.
- Sketches without labels or key values: the B marks are lost.
- Assuming a vehicle is still accelerating at the time found, without checking which stage it is in.

More tips

Memory tricks

- Rest-to-rest trapezium graph: distance $= \frac{1}{2} \times$ (time at top speed + total time) × top speed.
- Speed V reached from rest with acceleration a: time $\frac{V}{a}$, distance $\frac{V^2}{2a}$
- Check your answer: the total area of your sketch should equal the distance given in the question.
- Convert km to m before equating an area (in m) to a distance.

Exam technique

- A sketch needs the correct shape, labelled axes and the key values marked; it does not need to be to scale.
- Use the letters from the question (T, V) on your sketch and in your equations.
- If the question says 'use your graph', form equations from gradients and areas rather than using suvat.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Find velocity from a displacement–time graph**
The gradient of a displacement–time graph is the velocity.

Grade E — **Sketch a velocity–time graph from a description**
Use straight lines for constant acceleration and label the key speeds and times.

Grade D — **Find acceleration and distance from a velocity–time graph**
The gradient gives the acceleration and the area under the graph gives the displacement.

Grade C — **Use area to find an unknown time**
Write the total area in terms of the unknown and set it equal to the given distance.

Grade B — **Compare two objects on one graph**
Equal areas from the same start mean the objects are level; lines crossing only means equal velocities.

Grade A — **Handle graphs with negative velocity**
Area below the time axis is negative displacement; for the distance, add the sizes of all the areas.

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