Displacement, velocity and acceleration

M7.1 · Mechanics · revision notes

The language of motion: position, displacement, distance, velocity, speed and acceleration, and which of them are vectors. It comes up in short questions on average speed and average velocity, and in problems where position and velocity are given as $\mathbf{i}$, $\mathbf{j}$ vectors.

Key facts

- Vectors: displacement, velocity, acceleration. Scalars: distance, speed, time
- Average velocity = total displacement ÷ total time
- Angle with $\mathbf{i}$: $\tan\theta = \dfrac{b}{a}$
- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- Average speed = total distance ÷ total time
- Speed $= |\mathbf{v}| = \sqrt{a^2 + b^2}$ for $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$
- Constant velocity: $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

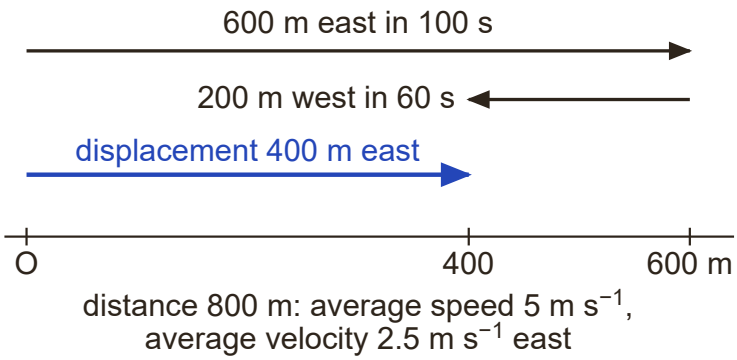
Notes

Scalars and vectors

- A **vector** has size and direction; a **scalar** has size only.
- **Position**: where an object is, relative to a fixed origin O. **Displacement**: the change in position (a vector).
- **Distance travelled**: the total length of the path (a scalar). It can never be negative.
- **Velocity**: rate of change of displacement (a vector). **Speed**: the magnitude of the velocity (a scalar, never negative).
- **Acceleration**: rate of change of velocity (a vector). An acceleration opposite to the direction of motion slows the object down: a **deceleration** or **retardation**.
- In a straight line, a sign shows the direction. Choose a positive direction and stick to it: a velocity of −3 m s^{-1} means a speed of 3 m s^{-1} in the negative direction.

Averages

- Average speed = total distance travelled ÷ total time taken.
- Average velocity = total displacement ÷ total time taken. It is a vector, so give its direction.

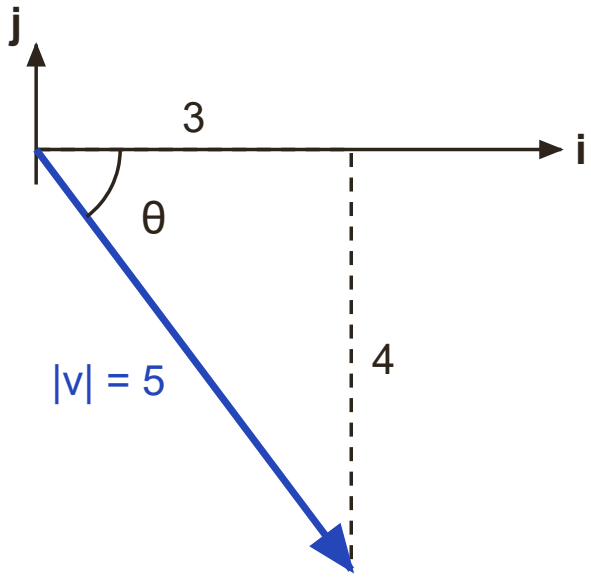


Distance counts every metre; displacement is just start to finish

- After a round trip the displacement is zero, so the average velocity is zero, but the average speed is not.
- An object moving at constant speed on a curved path has a changing velocity, because its direction changes.

Vectors in two dimensions

- $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors (often east and north). A position vector is $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$
- For a velocity $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, the speed is $|\mathbf{v}| = \sqrt{a^2 + b^2}$, and the angle with $\mathbf{i}$ comes from $\tan\theta = \dfrac{b}{a}$. Sketch the vector to get the right quadrant.



v = 3i − 4j: speed √(3² + 4²) = 5, at tan^{-1}(4/3) = 53.1° below i

- Constant velocity: $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, where $\mathbf{r}_0$ is the position when $t = 0$
- Displacement from A to B: $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$. The distance AB is its magnitude.
- With $\mathbf{i}$ east and $\mathbf{j}$ north, 'due north of O' means the $\mathbf{i}$ component is zero (and the $\mathbf{j}$ component positive); 'due east of O' means the $\mathbf{j}$ component is zero (and the $\mathbf{i}$ component positive).

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Distance, displacement and averages

3 to 5 marks | Grade E

1. Draw a quick line diagram and mark a positive direction.
2. Distance: add the lengths of all the parts. Displacement: the final position relative to the start, with a direction.
3. Divide by the total time, including any time spent at rest.

Example. A cyclist rides 600 m due east in 100 s and then rides 200 m due west in 60 s.
(a) Find the total distance travelled.
(b) Find the average speed of the cyclist.
(c) Find the average velocity of the cyclist.

Model answer.

(a) 600 + 200 = 800 m (B1)

(b) $\frac{800}{160}$ (M1) = 5 m s^{-1} (A1)

(c) Displacement = 600 − 200 = 400 m due east (M1); average velocity = $\frac{400}{160}$ = 2.5 m s^{-1} due east (A1)

Speed and position with a constant velocity vector

4 to 6 marks | Grade C

1. Speed is the magnitude of the velocity vector.
2. Write the position at time t as $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, collecting the $\mathbf{i}$ and $\mathbf{j}$ terms.
3. Turn the condition into an equation, e.g. 'due east of O' means the $\mathbf{j}$ component is zero.
4. Solve for t and check that it makes sense.

Example. [$\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north.] A boat moves with constant velocity $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}. At time $t = 0$ it is at the point with position vector $(2\mathbf{i} + 10\mathbf{j})$ m relative to a fixed origin O.
(a) Find the speed of the boat.
(b) Find the position vector of the boat at time t seconds.
(c) Find the value of t when the boat is due east of O

Model answer.

(a) $\sqrt{3^2 + (-4)^2}$ (M1) = 5 m s^{-1} (A1)

(b) $\mathbf{r} = (2\mathbf{i} + 10\mathbf{j}) + t(3\mathbf{i} - 4\mathbf{j})$ (M1) $= (2 + 3t)\mathbf{i} + (10 - 4t)\mathbf{j}$ m (A1)

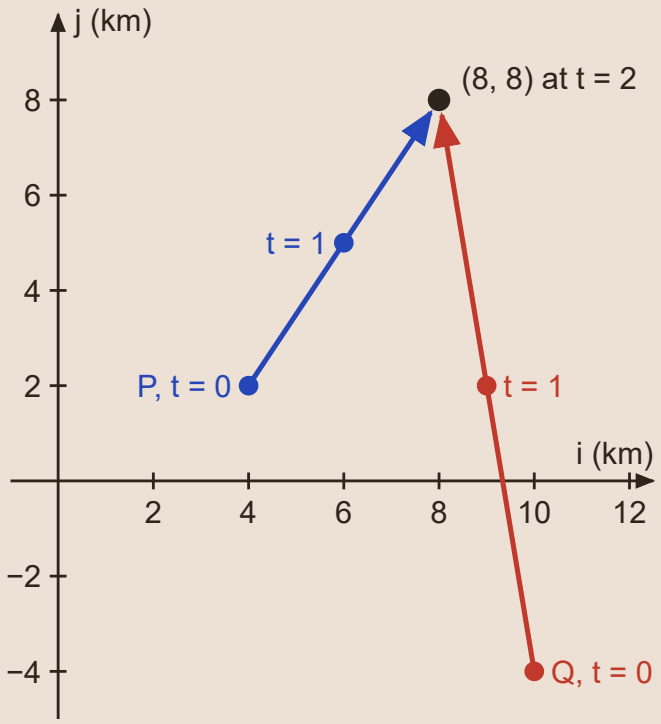
(c) $10 - 4t = 0$ (M1), so $t = 2.5$ (A1). The $\mathbf{i}$ component is then 9.5, which is positive, so the boat is east of O.

Two objects with constant velocities: do they meet? 5 to 7 marks **Grade A**

1. Write the position vector of each object at time t, using the same origin and the same t
2. They meet only if both the $\mathbf{i}$ and the $\mathbf{j}$ components are equal at the same time.
3. Solve using one component, then check that value of t in the other.
4. Finish with a conclusion giving the time and the position.

Example. [$\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north.] At noon, ship P is at the point with position vector $(4\mathbf{i} + 2\mathbf{j})$ km and ship Q is at the point with position vector $(10\mathbf{i} - 4\mathbf{j})$ km. P moves with constant velocity $(2\mathbf{i} + 3\mathbf{j})$ km h^{-1} and Q moves with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ km h^{-1}.
Show that, if neither ship changes its velocity, the ships will collide, and find the time and the position vector of the point where this happens.

Model answer.



t hours after noon: $\mathbf{p} = (4 + 2t)\mathbf{i} + (2 + 3t)\mathbf{j}$ and $\mathbf{q} = (10 - t)\mathbf{i} + (-4 + 6t)\mathbf{j}$ (M1 A1)

$\mathbf{i}$ components: $4 + 2t = 10 - t$, so $t = 2$ (M1 A1)

$\mathbf{j}$ components when $t = 2$: $2 + 6 = 8$ and $-4 + 12 = 8$, which are equal, so the ships collide (M1)

They collide at 2 pm at the point $(8\mathbf{i} + 8\mathbf{j})$ km (A1)

Don't lose marks

- Giving a negative distance or a negative speed.

- Using distance for average velocity, or displacement for average speed.
- Leaving out the starting position $\mathbf{r}_0$ in $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
- Giving an angle without saying what it is measured from (e.g. 'below $\mathbf{i}$', or as a bearing).
- Deciding that two objects meet after checking only one component.

More tips

Memory tricks

- Look out for Pythagorean triples in velocity vectors: 3, 4, 5; 5, 12, 13; 8, 15, 17; 7, 24, 25.
- 'Due north' → $\mathbf{i}$ component zero; 'due east' → $\mathbf{j}$ component zero; 'north-east' → equal positive components.
- Sketch a vector before finding an angle, so you know which quadrant it is in.
- Sense check: the average speed can never be less than the magnitude of the average velocity.

Exam technique

- Give a direction with every velocity or displacement, e.g. '2.5 m s^{-1} due east'.
- Keep vector answers in $\mathbf{i}$, $\mathbf{j}$ form unless a magnitude is asked for, and underline vectors in handwriting.
- In 'show that' questions write every substitution and finish with a clear statement.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Distinguish distance from displacement**
Displacement is the vector from the start to the finish; distance is the total length of the path.

Grade E — **Find average speed and average velocity**
Divide the total distance (for speed) or the total displacement (for velocity) by the total time.

Grade D — **Use signs to show direction in one dimension**
Choose a positive direction; a negative velocity means moving the opposite way at that speed.

Grade C — **Find speed and direction from a velocity vector**
Speed is $\sqrt{a^2 + b^2}$ for $a\mathbf{i} + b\mathbf{j}$; find the direction using $\tan\theta$ and a sketch.

Grade B — **Use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ for constant velocity**
Write the position vector at time t and use it to answer questions about where the object is.

Grade A — **Solve problems with two moving objects**
Write both position vectors in terms of t, then equate components or use the vector between them.

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